Multiple Access for UWB Impulse Radio With Pseudochaotic Time Hopping

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***Abstract*—Pseudochaotic time hopping (PCTH) is a recently proposed encoding/modulation scheme for ultra-wideband (UWB) impulse radio. PCTH exploits concepts from symbolic dynamics to generate aperiodic spreading sequences, resulting in a noise-like spectrum. In this paper, we present a multiple-access technique suitable for the PCTH scheme. In particular, we provide an analytical expression of the bit-error rate performance as a function of the number of users and validate it by simulation.**

***Index Terms*—Bit-error rate performances, chaos, indoor radio communication, multiaccess, time hopping, ultra-wideband.**

I. Introduction

OVER THE last decade, there has been a great interest in UWB impulse radio (IR) communication systems. These systems make use of ultra-short duration (<1 ns) pulses which yield ultra-wide bandwidth signals characterized by extremely low power spectral densities [1], [2]. UWB systems are particularly promising for short-range wireless communications as they potentially combine reduced complexity with low power consumption, low probability of intercept (LPI) and immunity to multipath fading. The successful deployment of the UWB technology depends strongly on the development of efficient multiple-access techniques. Existing UWB communication systems employ pseudorandom noise (PN) time hopping for multiple-access purposes, combined with pulse-position modulation (PPM) for encoding the digital information. An analysis of the multiuser capabilities of such systems has been presented by Scholtz *et al.* in [3]–[7].

Recently, it has been suggested to use aperiodic (chaotic) codes in order to enhance the spread-spectrum characteristics of UWB systems by removing the spectral features of the transmitted signal, thus resulting in a LPI. In addition, the absence of spectral lines may translate into a reduced interference toward other services such as GPS (global positioning system) [8]. In [9], the use of aperiodic sequences of pulses in the context of a chaos-based communication system was first proposed. A few schemes with chaotic modulation of the interpulse intervals were then studied in [10], [11]. In [12], a similar scheme was designed for the transmission of binary information and named chaotic pulse-position modulation (CPPM). Also, a scheme introducing a frequency modulation on top of the chaotic time hopping has been reported in [13].

In this paper, we consider a recently proposed modulation scheme for UWB IR, called psuedo chaotic time hopping (PCTH) [14], [15]. PCTH exploits concepts from symbolic dynamics [16] to generate aperiodic spreading sequences that, in contrast to fixed (periodic) PN sequences, depend on the input data. The PCTH scheme combines pseudochaotic encoding with a multilevel pulse-position modulation. The pseudochaotic encoder operates on the input data in a way that resembles a convolutional code [17]. Its output is then used to generate the time-hopping sequence resulting in a random distribution of the interpulse intervals, and thus a noise-like spectrum. Significant spreading demands for a large number of levels in the transmitter. This, in general, would require at the receiver side a convolutional decoder with a large number of states. In [15], it is shown that the PCTH signal can be decoded with a Viterbi detector [17] of reduced complexity, i.e., with a limited number of states.

We present a multiple-access extension to PCTH, that we call MA-PCTH. The basic idea consists of replacing each pulse transmitted by the original PCTH scheme with a pulse train, different for each user. The pulse train is a unique user "signature" very much like in code-division multiple-access (CDMA) schemes [18], but in the time domain. Each user is then demodulated with a filter matched to its signature.

This paper is organized as follows. In Section II, we briefly recall the operation of the PCTH scheme. Section III describes in detail the proposed multiple-access strategy for PCTH. Section IV contains the theoretical bit-error rate (BER) analysis of the MA-PCTH scheme. The corresponding simulation results are reported in Section V.

II. Pseudo-Chaotic Time Hopping

In this section, we recall the basics of the single-user PCTH scheme [15]. To this aim, we start by recalling some useful concepts from the shift map and its symbolic dynamics. Symbolic dynamics may be defined as a "coarse-grained" description of the evolution of a dynamical system [16]. The idea is to partition the state space and to associate a symbol to each partition.

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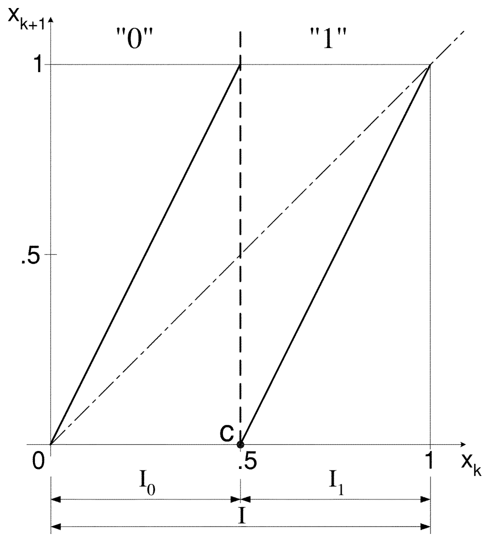

Fig. 1. The Bernoulli shift map with the definition of the symbolic dynamics used in the PCTH scheme. The invariant interval $I = [0, 1]$ is partitioned with respect to the critical point $c = 0.5$. The subintervals I_0 and I_1 are assigned the binary symbols "0" and "1," respectively.

Consequently, a trajectory of the dynamical system can be analyzed as a symbolic sequence. A simple example of a chaotic map is the *Bernoulli shift* [19], defined as

$$x_{k+1} = 2x_k \mod 1 \tag{1}$$

whose graph is shown in Fig. 1. The state x can be expressed as a binary expansion

$$x = 0.b_1b_2b_3\ldots = \sum_{j=1}^{\infty} 2^{-j}b_j \tag{2}$$

with b_j equal to either "0" or "1," and $x \in I = [0, 1]$. For this map, a Markov partition [16] can be selected by splitting the interval $I = [0, 1]$ into two subintervals: $I_0 = [0, 0.5)$ and $I_1 = [0.5, 1]$. Then, in order to obtain a symbolic description of the dynamics, the binary symbols "0" and "1" are associated with the subintervals I_0 and I_1, respectively.

In PCTH, the Bernoulli shift (1) is approximated by means of a finite-length (M-bit) shift register, R. Multiplication by two in (1) corresponds to a left shift (b_2 goes to b_1, etc.), while the modulo one operation is realized by discarding the most significant bit (MSB). At each clock impulse the most recent bit of information is assigned the least significant bit (LSB) position in the shift register, while the old MSB is discarded. From the viewpoint of information theory, the shift register implementing the Bernoulli shift may be seen as a form of convolutional coding [16]. The memory of the structure is given by the shift register which stores the last M input bits. Each input bit causes an output of M bits; thus, the overall rate is $1/M$. In general, the shift register may be followed by a transformation unit for generating more complex chaotic maps. For example, the simple transformation operated by a Gray/binary converter on the Bernoulli shift leads to the *tent map*, described by

$$x_{k+1} = 1 - 2|x_k - 0.5|. \tag{3}$$

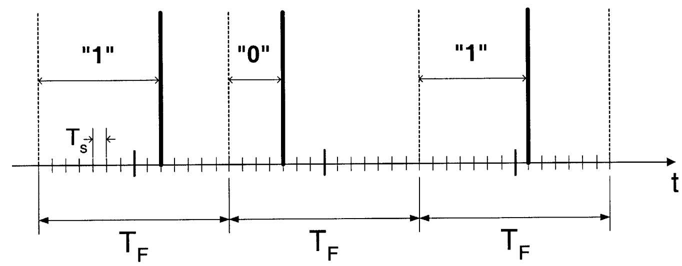

Fig. 2. Sketch of the periodic frame. The frame time and the timeslot associated with each PPM level are denoted by T_F and T_s, respectively.

In the PCTH scheme, the output of the pseudochaotic encoder is used to drive a pulse-position modulator. Namely, each pulse is allocated, according to the pseudochaotic modulation, within a periodic frame of period T_F, as shown schematically in Fig. 2. In other words, only one pulse is transmitted within each symbol period T_F. If the pulse occurs in the first half of the frame a "0" is being transmitted, otherwise a "1." Each pulse can occur at any of $N = 2^M$ discrete time instants, where M is the number of bits in the shift register R. In Fig. 2, the timeslot duration corresponding to each possible level of the pulse-position modulation has been denoted by T_s.

The PCTH receiver comprises a pulse correlator, matched to the pulse-shape, followed by a pulse-position demodulator (PPD) and a detector. In the simplest case, the binary information may be retrieved by means of a threshold discriminator at the output of the PPD. For the sake of simplicity, in this work we will not consider maximum-likelihood (ML) sequence estimation using the Viterbi algorithm, as in [15]. Instead, the detection will rely on a simple ML criterion based on the observation of a single frame.

III. Multiple Access for PCTH

Fig. 3 shows a simplified block diagram for the proposed multiple-access scheme based on pseudochaotic time hopping, that we denote by MA-PCTH. The transmitter/receiver architecture shown in Fig. 3 refers to the generic jth user. The input to the system is an independent identically distributed (i.i.d.) source of binary data, $b_k^{(j)}$, where the lower index denotes the kth bit. The input sequence feeds the pseudochaotic encoder, whose operation has been described in Section II. As in PCTH, the output of the pseudochaotic encoder, $d_k^{(j)}$, drives the N-PPM modulator, thus producing the time hopping. In MA-PCTH, though, the output of the modulator is used to trigger a pulse train generator corresponding to the specific signature, $c^{(j)}$, associated with the jth user. In this paper, for simplicity, we consider a slotted system where the periodic frames (of period T_F) corresponding to the different users are synchronized with each other. Each frame is subdivided into N slots of duration $T_s (= T_F/N)$. In turn, each slot contains N_c chips; correspondingly, the chip time is given by $T_c = T_s/N_c$. In our analysis, we assume that for each user the pulse trains are confined within the slot time T_s, i.e., the user signatures do not invade adjacent slots. This also implies that, within a given frame T_F, two generic users (j) and (k) will either transmit in different slots or collide. The situation, for a single frame period, is illustrated in Fig. 4.

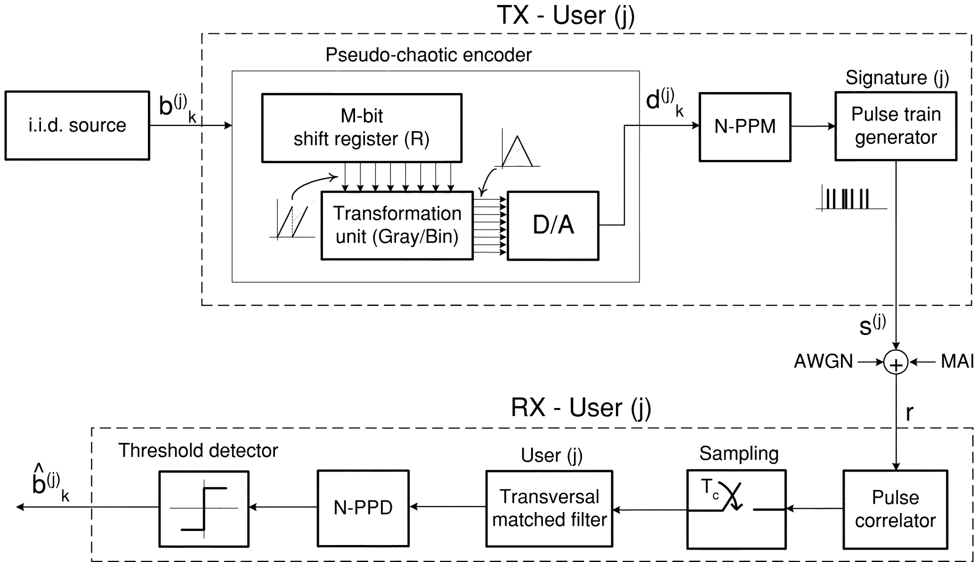

Fig. 3. Simplified block diagram of the MA-PCTH scheme.

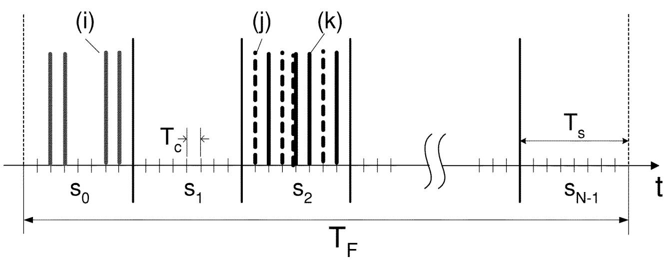

Fig. 4. Sketch of the periodic frame for the MA-PCTH scheme with three users. The frame period, T_F, is divided into N slots, each of duration $T_s(=T_F/N)$. Note the different "signatures" associated with the different users. Users (j) and (k) exhibit a collision in the third slot (s_2).

The transmitted signal, $s^{(j)}(t)$ for the jth user can be expressed for each frame as

$$s^{(j)}(t) = \sum_{l=0}^{N_c-1} c_l^{(j)} w_p\left(t - lT_c - d_k^{(j)}T_s\right), \qquad \times t \in [0, T_F), \quad k = 0,1,2,\ldots \quad (4)$$

where $c_l^{(j)} \in \{0,1\}$ ($l = 0,\ldots,N_c-1$) is the binary sequence representing the jth user signature. On the other hand, $w_p(t)$ is the pulse waveform that in this work is assumed to be rectangular

$$w_p(t) = \begin{cases} 1, & 0 < t < t_p \\ 0, & \text{otherwise} \end{cases}$$

where t_p is the pulse duration and $t_p < T_c$. So, for each information bit, $b_k^{(j)}$, a pseudochaotic iterate $d_k^{(j)} \in \{0,\ldots,N-1\}$ is generated and the pulse train for the jth user is transmitted in the corresponding slot, within the frame time.

In the case of a single user, the signal received is simply

$$r(t) = s(t) + n(t)$$

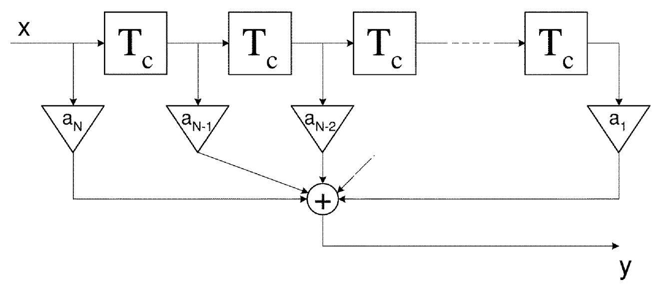

Fig. 5. Transversal matched filter.

where $n(t)$ is additive white Gaussian noise (AWGN). In general, with N_u users transmitting simultaneously, the input to the jth receiver will be

$$r(t) = s^{(j)}(t) + n_u^{(j)}(t) + n(t)$$

where the term $n_u^{(j)}(t)$ takes into account the multiple-access interference (MAI) caused by the remaining $(N_u - 1)$ users sharing the channel.

Referring to Fig. 3, the jth receiver comprises a pulse correlator for the pulse waveform $w_p(t)$. The output of the correlator is given by

$$\rho_{si} = \int_{iT_c+sT_s}^{(i+1)T_c+sT_s} w_p(\tau)r(\tau)d\tau, \quad i = 0,\ldots,N_c-1$$

which is sampled according to the chip time T_c. The samples ρ_{si} are then fed into a (digital) transversal matched filter implemented by a tapped delay line [20], whose architecture is illustrated in Fig. 5. In the case under consideration, the weights a_i should coincide with the user signature, that is

$$a_i \equiv c_i^{(j)}, \quad i = 0,\ldots,N_c-1.$$

Thus, the output of the matched filter for slot s is

$$y_s^{(j)} = \sum_{i=0}^{N_c-1} c_i^{(j)} \rho_{si}, \quad s = 0,\ldots,N-1 \quad (5)$$

where the subscript s runs over the number of slots per frame. The pulse-position demodulation is carried out by applying a ML criterion (that is selecting the largest sample at the output of the matched filter) for each frame period T_F. Namely, the most likely slot, $\hat{s}^{(j)}$, is

$$\hat{s}^{(j)} = \arg\max_s \left\{ y_s^{(j)}, \quad s = 0, \ldots, N-1 \right\}$$

Finally, the estimate $\hat{b}_k^{(j)}$ of the transmitted bit (for the jth user) can be obtained by means of a threshold detector.

IV. BER Performance: Theoretical Analysis

In this section, we analyze the BER performance of the MA-PCTH scheme in the presence of AWGN. The signal-to-noise ratio (SNR) is defined by E_b/N_0, where E_b is the energy per user bit and N_0 is the single-sided spectral noise power density ($\sigma_n^2 = N_0/2$) of the AWGN. We assume that the receiver tries to demodulate the data transmitted by user 1 in the presence of MAI introduced by the ($N_u - 1$) other users. The cross correlation value with each user is normalized to the autocorrelation value of user 1. We present a detailed analysis for the two- and three-user case, and provide a general BER expression for an arbitrary number of users. The baseline behavior being represented by the (uncoded) single-user PCTH scheme. We emphasize that for a single user the BER performance of PCTH coincides with orthogonal N-ary PPM.

A. Two-User Case

For each frame, the two users transmit either in a different slot or in the same slot. By denoting these mutually exclusive events with A and B, respectively, then the probability of detecting user 1 in the wrong slot, P_e, is given by

$$P_e = P(\text{error} \mid A)P(A) + P(\text{error} \mid B)P(B)$$

where $P(A) = (N-1)/N$ and $P(B) = 1/N$ denote the probability of A and B, respectively, and $P(\text{error} \mid A)$ and $P(\text{error} \mid B)$ are the probabilities of error given events A and B), respectively. $P(\text{error} \mid A)$ is obtained by modifying the symbol error probability of N-ary orthogonal signaling [17] by considering that user 1 and user 2 do not transmit in the same slot

$$P(\text{error} \mid A) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [1 - \Phi(y - \sqrt{2S_1}\gamma_2) \times \Phi(y)^{N-2}] e^{\frac{-(y-\sqrt{2S_1})^2}{2}} \, dy \quad (6)$$

where $\Phi(x) = (1/\sqrt{2\pi}) \int_{-\infty}^{x} e^{-(t^2/2)} \, dt, S_1 = E_1/N_0$ is the SNR of user 1, with E_1 the transmitted energy of user 1, and $\gamma_2 = \sum_{i=0}^{N_c-1} c_i^{(1)} c_i^{(2)}$ is the periodic cross correlation between user 2 and user 1, the user of interest. Similarly, $P(\text{error} \mid B)$ is obtained by considering that user 1 and user 2 transmit in the same slot

$$P(\text{error} \mid B) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [1 - \Phi(y)^{N-1}] \times e^{-\frac{(y-\sqrt{2S_1}(1+\gamma_2))^2}{2}} \, dy. \quad (7)$$

B. Three-User Case

If three users are present, all three can transmit in different slots (event denoted by A), all three can transmit in the same slot (event B), or each of the three possible pairs of users can transmit in the same slot (events C_{12}, C_{23}, C_{13}). Specifically, C_{ij} corresponds to users i and j transmitting in the same slot and the remaining user in a different slot. The average error probability, P_e, of detecting user 1 in the wrong slot is given by

$$\begin{aligned} P_e = {} & P(\text{error} \mid A)P(A) + P(\text{error} \mid B)P(B) \\ & + P(\text{error} \mid C_{12})P(C_{12}) + P(\text{error} \mid C_{23})P(C_{23}) \\ & + P(\text{error} \mid C_{13})P(C_{13}). \end{aligned}$$

Assuming all users transmit i.i.d. binary data, $P(A) = (N-1)(N-2)/N^2, P(B) = 1/N^2$ and $P(C_{12}) = P(C_{23}) = P(C_{13}) = (N-1)/N^2$. $P(\text{error} \mid A)$ is obtained by modifying the symbol error probability of N-ary orthogonal signaling [17], by considering that users 1, 2, and 3, each transmit in different slots

$$P(\text{error} \mid A) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [1 - \Phi(y - \sqrt{2S_1}\gamma_2) \times \Phi(y - \sqrt{2S_1}\gamma_3)\Phi(y)^{N-3}] e^{-\frac{(y-\sqrt{2S_1})^2}{2}} \, dy \quad (8)$$

where $\gamma_j = \sum_{i=0}^{N_c-1} c_i^{(1)} c_i^{(j)}$ denotes the periodic cross correlation between user 1 and j. Note that increasing γ_j decreases the value of Φ in the integrand, thus, increases the error probability. On the other hand, $P(\text{error} \mid B)$ is obtained by considering that the interference due to user 2 and user 3 appears in the same slot occupied by user 1

$$P(\text{error} \mid B) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [1 - \Phi(y)^{N-1}] \times e^{-\frac{(y-\sqrt{2S_1}(1+\gamma_2+\gamma_3))^2}{2}} \, dy. \quad (9)$$

The effect of users 2 and 3 transmitting in the same slot as user 1 can be readily seen from (9) as effectively improving the SNR and decreasing the error probability.

The probability of error for the remaining events C_{12}, C_{23}, and C_{13} can be calculated using

$$P(\text{error} \mid C_{ij}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [1 - \Phi(y - \sqrt{2S_1}\gamma_d) \times \Phi(y)^{N-2}] e^{-\frac{(y-\sqrt{2S_1}(1+\gamma_s))^2}{2}} \, dy \quad (10)$$

where γ_d denotes the total cross correlation of users transmitting in the same slot, but different from the slot occupied by user 1, and γ_s indicates the total cross correlation of users occupying the same slot as user 1. Then, $P(\text{error} \mid C_{12})$ is found by setting $\{\gamma_d = \gamma_3, \gamma_s = \gamma_2\}, P(\text{error} \mid C_{13})$ is found by setting $\{\gamma_d = \gamma_2, \gamma_s = \gamma_3\}$, while $P(\text{error} \mid C_{23})$ can be obtained by setting $\{\gamma_d = \gamma_2 + \gamma_3, \gamma_s = 0\}$.

C. General Case

For N_u users we generalize the previous considerations to the following interference event denoted by C. There are n slots indexed by $i = 1, \ldots, n$, different from the slot used by user 1, and slot i contains α_i interfering users. The slot occupied by user 1 receives contributions from $N_u - 1 - \sum_{i=1}^{n} \alpha_i$ interferers and all the others slots are not used. The probability that user 1 is detected in the wrong slot is given by (11), as shown at the bottom of the page [21], where $\gamma^{(i,k)}$ represents the cross correlation between user 1 and the interferer indexed by k, in the slot indexed by i, while $\gamma^{(k)}$ is the cross correlation between the interfering user indexed by k and user 1.

In order to calculate the average probability of error in the general case, we need an expression for the probability of each the possible interference events. The average probability of error is

$$P_e = P(\text{error} \mid A)P(A) + P(\text{error} \mid B)P(B) + P(C'). \quad (12)$$

A denotes the event where all users transmit in the same slot, B is the event where all users transmit in different slots, and C' denotes the collection of all other interference events. It follows that: $P(A) = (N-1)(N-2)\ldots(N-(N_u-1))/N^{N_u-1}$, and $P(B) = 1/N^{N_u-1}$. Moreover

$$P(\text{error} \mid A) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[1 - \Phi(y)^{N-N_u} \times \prod_{i=2}^{N_u} \Phi(y - \sqrt{2S_1}\gamma_j)\right] e^{-\frac{(y-\sqrt{2S_1})^2}{2}} \, dy \quad (13)$$

$$P(\text{error} \mid B) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [1 - \Phi(y)^{N-1}] \times e^{-\frac{\left(y - \sqrt{2S_1}\left(1 + \sum_{j=2}^{N_u} \gamma_j\right)\right)^2}{2}} \, dy \quad (14)$$

where again γ_j denotes the periodic cross correlation between user 1 and j. In addition

$$P(C') = \sum_{C \in \mathcal{C}} P(\text{error} \mid C)P(C) \quad (15)$$

where $\mathcal{C}$ indicates the set of all interference events, except A and B. In practice, if the user signatures are equicorrelated, $P(C')$ can be calculated as the weighted average of the probability of error of all events in $\mathcal{C}$, as shown in (16), at the bottom of the page, where, $a_0 = \min(N_u - 1, N_u - \sum_{i=2}^{\lambda} a_i)$. β_2 is the number of slots in which two users transmitted, and β_3 is the number of slots in which three users transmitted, etc. In (16), $\lambda_{\max} = \lfloor (N_u/2) \rfloor$ is the maximum number of different possible interference events within a single frame. This occurs when $(N_u/2)$ pairs of users interfere. The number of users who transmit in slots with no interfering users is $\alpha_0 = N_u - \sum_{i=1}^{\lambda} a_i$. In the above expression, the events $A_1, \ldots, A_\lambda$ correspond to a_1 users transmitting in the same slot, a_2 users transmitting in the same slot, but different than the A_1 users, etc. The superscript 1 indicates that user 1, the user of interest is included in that set.

For all cases, in order to calculate the BER, we need to convert symbol errors to bit errors. Namely, we convert the probability of detecting user 1 in a wrong slot, P_e, to the bit error probability P_b. The errors which consist of confusing the slot used by user 1 with any of the other $N-1$ slots are equiprobable and occur with probability

$$\frac{P_e}{N-1} = \frac{P_e}{2^M - 1}$$

Let us assume without loss of generality that the binary information digit transmittedd by user 1 is zero; then the probability that the receiver makes a bit error is the probability of confusing the slot where user 1 is transmitting with any of the last $N/2$ slots in the frame. Thus

$$P_b = \frac{2^{M-1}}{2^M - 1} P_e \approx \frac{P_e}{2}, \quad M \gg 1. \quad (17)$$

D. Example: Four-User Case

The probability of detecting user 1 in the wrong slot can be subdivided into the following cases.

1) The four users transmit in the same slot, with probability $1/N^3$.
2) All users transmit in different slots, with probability $(N-1)(N-2)(N-3)/N^3$.

$$P(\text{error} \mid C) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[1 - \prod_{i=1}^{n} \Phi\left(y - \sqrt{2S_1} \sum_{k=1}^{\alpha_i} \gamma^{(i,k)}\right) \times \Phi(y)^{N-1-n}\right] \cdot \exp\left[-\frac{1}{2}\left(y - \sqrt{2S_1}\left(1 + \sum_{k=1}^{N_u - 1 - \sum_{i=1}^{n} \alpha_i} \gamma^{(k)}\right)\right)^2\right] dy \quad (11)$$

$$P(C') = \frac{1}{N^{N_u-1}} \sum_{\lambda=1}^{\lambda_{\max}} \sum_{a_\lambda=2}^{a_{\lambda-1}} \cdots \sum_{a_3=2}^{a_2} \sum_{a_2=2}^{a_1} \sum_{a_1=2}^{a_0} \binom{N_u}{a_1} \times \binom{N_u - a_1}{a_2} \ldots \binom{N_u - a_1 - a_2 - \cdots - a_{\lambda-1}}{a_\lambda} \cdot (N-1)(N-2)\ldots\left(N - \left(N_u - \sum_{i=1}^{\lambda-1} a_i\right) - (\lambda - 1)\right)\left(\frac{1}{\beta_2!\beta_3!\ldots}\right)\frac{1}{N_u} \cdot \left[a_1 Pr_e\left(A_1^1, A_2, \ldots, \alpha_0\right) + a_2 Pr_e\left(A_1, A_2^1, \ldots, \beta_0\right) + \cdots + \alpha_0 Pr_e\left(A_1, A_2, \ldots, \alpha_0^1\right)\right] \quad (16)$$

3) Two users transmit in the same slot and the two remaining users transmit in independently different slots, with probability $(N-1)(N-2)/N^3$.
4) Three users transmit in the same slot and the remaining user transmits in a different slot, with probability $(N-1)/N^3$.
5) Two users transmit in the same slot and the two remaining users transmit in the same slot different from the previous two, with probability $(N-1)/N^3$.

If the cross correlation between user 1 and all other users is not equal, then, each of the above events must be subdivided further. For instance, the probability that two users transmit in the same slot and the two remaining users transmit in the same slot, different from the previous two (case 5), is the sum of the probabilities that:

5a User 1 and 2 transmit in the same slot and user 3 and 4 transmit in the same slot different from the other users, with probability $(1/3)(N-1)/N^3$.

5b User 1 and 3 transmit in the same slot and user 2 and 4 transmit in the same slot different from the other users, with probability $(1/3)(N-1)/N^3$.

5c User 1 and 4 transmit in the same slot and user 2 and 3 transmit in the same slot different from the other users, with probability $(1/3)(N-1)/N^3$.

The probability of error for each of these events can be evaluated by applying (11). For example, the probability of case (5a) is obtained by setting $n=1$, $\alpha_1=2$, and $\gamma^{(1,1)}$ being the cross correlation between users 1 and 3, and $\gamma^{(1,2)}$) being the cross correlation between users 1 and 4. $\gamma^{(1)}$ denotes the cross correlation between users 1 and 2.

V. Simulation Results

This section reports the simulation results for the MA-PCTH scheme and compares them with the theoretical predictions. The results of our analysis are presented in terms of BER probability versus the SNR at the receiver, expressed in decibels.

Fig. 6 shows the simulated and analytical BER for single-user PCTH. The analytical calculation uses the method outlined in Section IV. We used $M=8$ bits corresponding to $N=256$ PPM levels, with $N_c = 32$ chips/slot. As mentioned in Section IV, the BER performance coincides with orthogonal 256-PPM. This is the baseline from which to compare the multiuser cases since it represents the best performance that could be possibly achieved if, for example, the various users had orthogonal signature sequences (zero cross correlation).

In each of the multiuser cases, a unique 32-bits signature sequence was assigned to each user. The binary sequences that we chose to use were randomly selected. The only constraint imposed on the sequence selection process was that each sequence should contain an equal number of ones (specifically 16 ones and 16 zeros). This maintains a constant energy across all users. The randomly selected sequences have periodic cross correlation values to user 1, the user of interest, of 0.3750, 0.4375, and 0.5625. The family of curves in Fig. 7 shows the two-user simulated performance with each of these cross-correlation values. Note how the BER performance improves with decreasing cross correlation [see (6)]. As the cross correlation

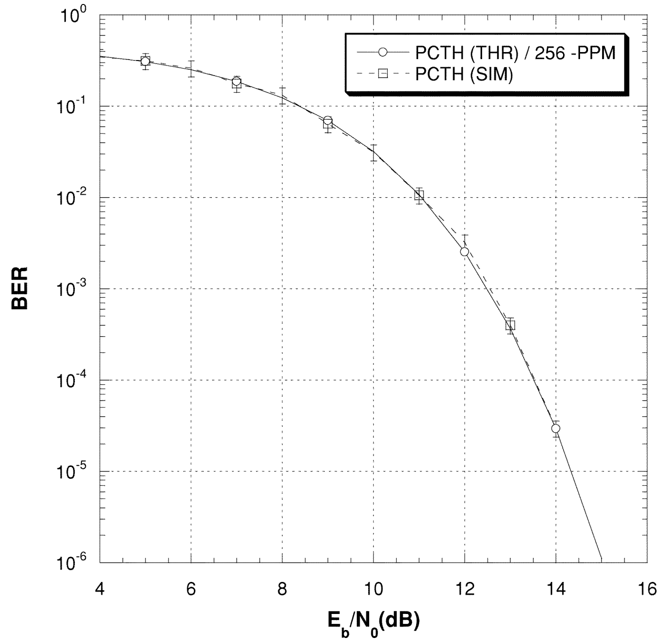

Fig. 6. Simulated versus analytical BER performance of the single-user PCTH scheme in the presence of AWGN. Note that in this case the error probability coincides with orthogonal 256-PPM.

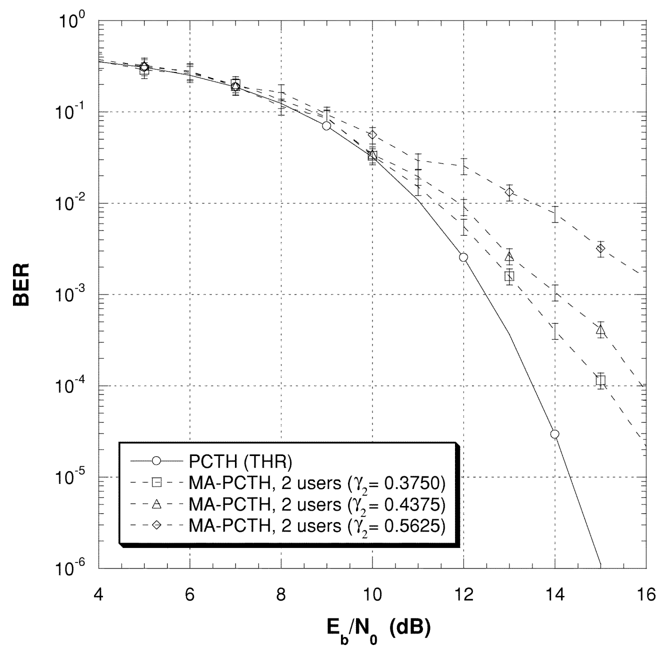

Fig. 7 Simulated BER performance of the MA-PCTH scheme with two users, for various values of cross correlation.

of the second user's sequence increases, the probability of choosing the slot in which user 2 transmitted increases, and so does the intended user's probability of error. This is consistent with the fact that orthogonal signaling results in the best possible BER performance. In Fig. 8, we show a comparison of the simulated performance versus the theoretical predictions, for each cross-correlation value.

Fig. 9 shows the analytical BER performance, using (11), and simulated performance of four users. The three interfering users had cross-correlation values to the first user of 0.3750, 0.4375,

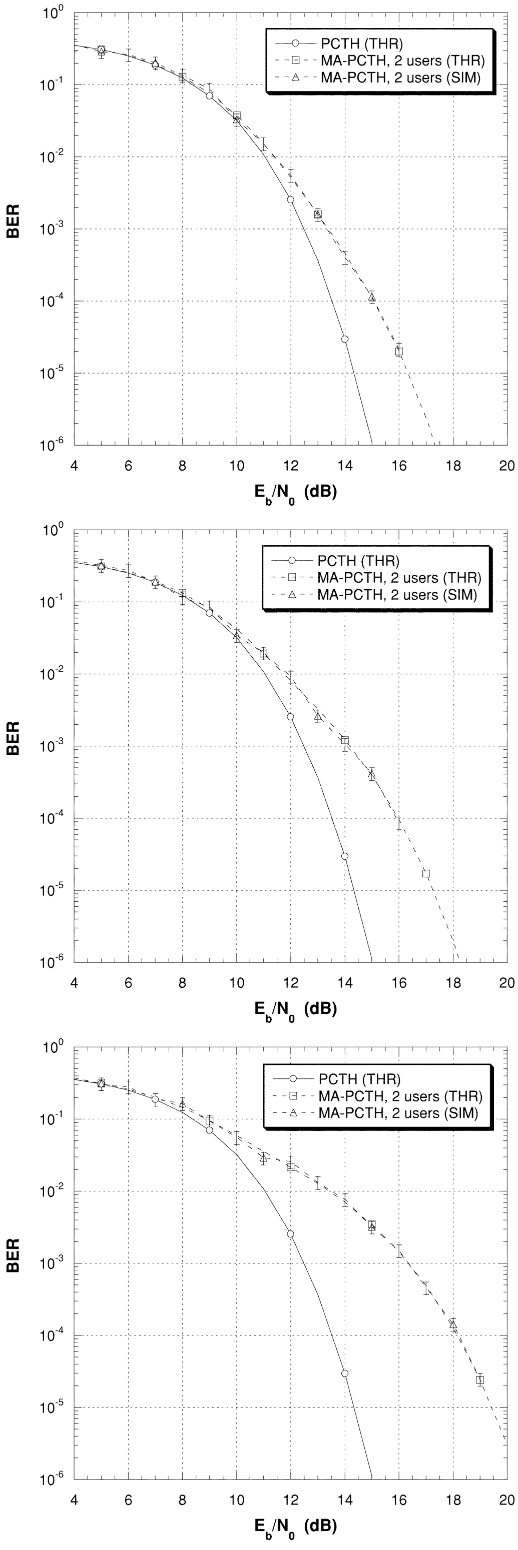

Fig. 8. Simulated versus analytical BER performance of the MA-PCTH scheme with two users, for various values of cross correlation.

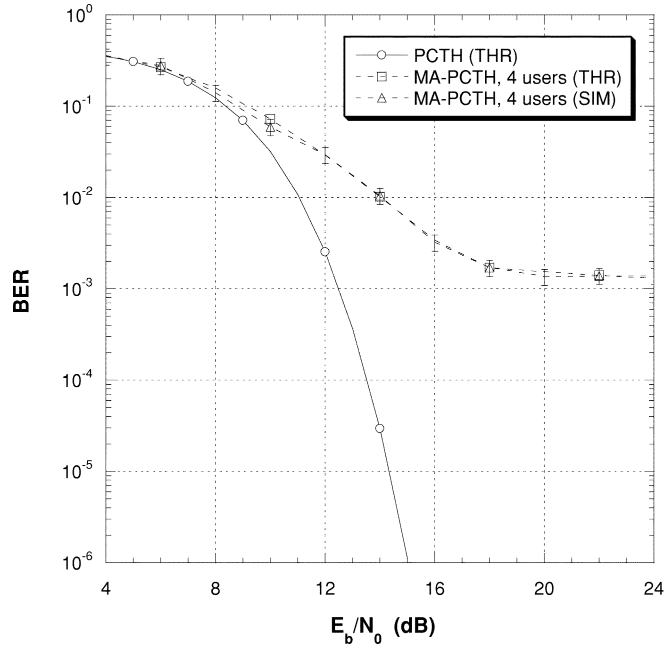

Fig. 9. Simulated versus analytical BER performance of the MA-PCTH scheme with four users. Users 2, 3, 4 have cross-correlation values of 0.3750, 0.4375, and 0.5625, respectively, to user 1. Note that with enough users, an error floor in the BER can develop.

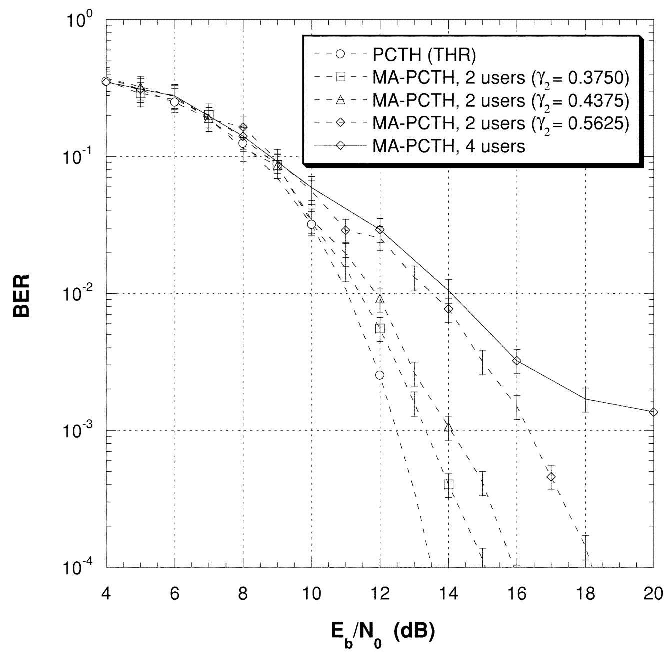

Fig. 10. Simulated BER dependence on the cross correlation. For the four-user case the cross-correlation values are the same as in Fig. 9.

and 0.5625. Note that, as pointed out in [22], depending on the value of the cross correlation and/or with enough users an error floor in the BER can develop. Fig. 10 shows the simulated performance of the same four-user case compared with each of the two-user cases previously discussed. The four-user system performance is dominated by the user with the highest cross correlation.

Finally, Fig. 11 shows the BER performance of the simulated system as a function of the number of users, N_u. In the simulated two-user data, the sequence with cross correlation of 0.5625 was used. For the four-user case the cross-correlation values were set to 0.3750, 0.4375, and 0.5625, with respect to user 1. In the eight-user case, we assigned a cross-correlation value of 0.3750

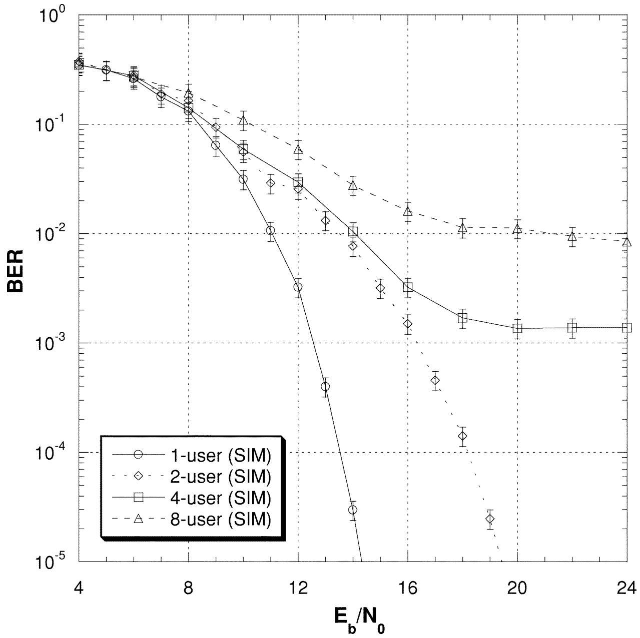

Fig. 11. Simulated BER dependence on the number N_u of users. In the two-user case, the cross correlation equals 0.5625. In the four-user case, the cross-correlation values are 0.3750, 0.4375, and 0.5625, with respect to user 1. In the eight-user case, two users have cross correlation 0.3750, three other users 0.4375, and the remaining two interfering users 0.5625.

to two users, a value of 0.4375 to three other users, and 0.5625 to the remaining two interfering users. As the number of users increases, the BER performance is degraded. This is true provided that the cross correlation increases with an increasing number of users, as in our case.

VI. Conclusion

The success of MA-PCTH as a communication system depends on how many users can be supported at a sufficiently low error rate and a sufficiently high data rate. In this paper, we have shown that the BER performance of single-user PCTH is the same as N-ary orthogonal signaling (e.g., N-PPM). For multiuser communications, there are many parameters that need to be optimized in order to produce the best overall system performance. These include the length of the signature sequences, the cross correlation between sequences, the dimensionality of the pulse-position modulator and the system data rate capacity. In this work, we have investigated the influence of the periodic cross correlation between the generic user and each of the other users on the BER, in a synchronous system. Our analysis indicates that the highest cross correlation amongst interferers tends to dominate the BER performance. For future work, in order to improve the BER performance, one needs to find sets of signature sequences exhibiting constant cross correlation between any two sequences in the set, or that are bounded by an acceptable level. An interesting variation of the proposed scheme is the nonslotted MA-PCTH case, where $NcTc > Ts$. In this case, the length of each user's sequence extends through one slot into an adjacent slot causing intrasymbol interference. When users interfere with each other, they no longer do it in a unique way because there is more than one way that the sequences can overlap. Aperiodic cross correlations must now be considered. The possible benefits are a potentially improved data rate, simpler implementation, and the potential to support more users, albeit with a possibly higher BER.

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References

[1] M. Z. Win and R. A. Scholtz, "Impulse radio: How it works,," *IEEE Commun. Lett.*, vol. 2, pp. 36–38, Feb. 1998.
[2] S. S. Kolenchery, J. K. Townsend, and J. A. Freebersyer, "A novel impulse radio network for tactical military wireless communications," in *Proc. MILCOM*, Boston, MA, Oct., 18–21 1998, pp. 59–65.
[3] R. A. Scholtz, "Multiple access with time hopping impulse modulation," in *Proc. MILCOM*, Bedford, MA, Oct. 11–13, 1993.
[4] F. Ramirez-Mireles and R. A. Scholtz, "N-orthogonal time-shift-modulated signals for ultra-wide bandwidth impulse radio modulation," *Proc. IEEE Commun. Theory Mini Conf.*, Nov. 1997.
[5] M. Z. Win and R. Scholtz, "Impulse radio: How it works," *IEEE Commun. Lett.*, vol. 2, pp. 36–38, Feb. 1998.
[6] F. Ramirez-Mireles and R. A. Scholtz, "Multiple-access performance limits with time hopping and pulse-position modulation," in *Proc. MILCOM'98*, vol. 2, Oct. 1998, pp. 529–533.
[7] M. Z. Win and R. A. Scholtz, "Ultra-wide bandwidth time-hopping spread-spectrum impulse radio for wireless multiple access communications," *IEEE Trans. Commun.*, vol. 48, pp. 679–689, Apr. 2000.
[8] R. G. Aiello, G. D. Rogerson, and P. Enge, "Preliminary assessment of interference between ultra-wideband transmitters and the global positioning system: A cooperative study," presented at the National Technical Meeting of the Institute of Navigation, Jan. 2000.
[9] P. A. Bernhardt, "Chaotic frequency modulation," in *Proc. SPIE—Int. Society Optical Engineering*, vol. 2038, 1993, pp. 162–181.
[10] N. F. Rulkov and A. R. Volkovskii, "Threshold synchronization of chaotic relaxation oscillations," *Phy. Lett. A*, vol. 179, no. 4–5, pp. 332–336, 1993.
[11] H. Torikai, T. Saito, and W. Schwarz, "Synchronization via multiplex pulse trains," *IEEE Trans. Circuits and Systems—I*, vol. 46, pp. 1072–1085, Sept. 1999.
[12] M. Sushchick, N. Rulkov, L. Larson, L. Tsimring, H. Abarbanel, K. Yao, and A. Volkovskii, "Chaotic pulse position modulation: A robust method of communicating with chaos," *IEEE Commun. Lett.*, vol. 4, pp. 128–130, Apr. 2000.
[13] T. Yang and L. O. Chua, "Chaotic impulse radio: A novel chaotic secure communication system," *Int. J. Bifurcation Chaos*, vol. 10, pp. 345–357, Feb. 2000.
[14] G. M. Maggio, N. Rulkov, M. Sushchik, L. Tsimring, A. Volkovskii, H. Abarbanel, L. Larson, and K. Yao, "Chaotic pulse-position modulation for ultrawide-band communication system," in *UWB'99*, Washington, D.C., Sept. 28-30, 1999.
[15] G. M. Maggio, N. Rulkov, and L. Reggiani, "Pseudo-chaotic time hopping for UWB impulse radio," *IEEE Trans. Circuits Syst.—I*, vol. 48, pp. 1424–1435, Dec. 2001.
[16] D. Lind and B. Marcus, *An Introduction to Symbolic Dynamics and Coding*. Cambridge, U.K.: Cambridge Univ. Press, 1995.
[17] J. G. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995.
[18] A. J. Viterbi, *CDMA Principles of Spread Spectrum Communication*. Reading, MA: Addison-Wesley, 1995.
[19] E. Ott, *Chaos in Dynamical Systems*. Cambridge, U.K.: Cambridge Univ. Press, 1993.
[20] L. W. Couch, *Digital and Analog Communication Systems*. Englewood Cliffs, NJ: Prentice-Hall, 1997.
[21] G. M. Maggio, D. Laney, F. Lehmann, and L. Larson, "A multi-access scheme for UWB radio using pseudochaotic time hopping," in *Proc. UWBST 2002*, Baltimore, MD, May. 20–23, 2002, pp. 225–229.
[22] G. M. Maggio, D. Laney, and L. Larson, "BER for synchronous multi-access UWB radio using pseudochaotic time hopping," in *Proc. GLOBECOM 2002*, Taipei, Taiwan, Nov. 17–21, 2002, to be published.

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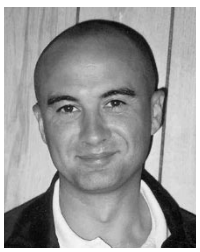

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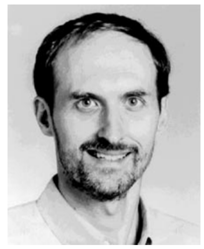

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