BER for Multi-Access UWB Radio Using Pseudo-Chaotic Time Hopping

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***Abstract*— Pseudo-chaotic time hopping (PCTH) is a recently proposed modulation scheme for UWB impulse radio. PCTH exploits concepts from symbolic dynamics to generate aperiodic spreading sequences. In this paper, we present a general analytical expression for the average bit error rate (BER) performance of a synchronous multi-access PCTH system as a function of the cross-correlation between the users' signatures in an additive white Gaussian noise (AWGN) channel. Also, it is shown that with enough users an error floor in the BER can develop.**

I. INTRODUCTION

Over the last decade, there has been a great interest in communication based on impulse radio. These systems make use of ultra-short duration pulses which yield ultra-wide band (UWB) signals characterized by low power spectral densities [1], [2]. UWB systems are particularly promising for short-range wireless communications as they potentially combine reduced complexity with low power consumption, low-probability-of-intercept (LPI) and immunity to multipath fading. The successful deployment of UWB technology depends strongly on the development of efficient multi-access techniques. Existing UWB communication systems employ pseudo-random noise (PN) time hopping for multi-access purposes combined with pulse-position modulation (PPM) for encoding the digital information. An analysis of the multi-user capabilities of such systems has been presented by Scholtz *et al.* in [3], [4], [5].

Recently, it has been suggested that aperiodic codes may be used to reduce the spectral features of the transmitted signal. In this work, we consider an extension of the pseudo-chaotic time hopping (PCTH) scheme described in [6]. PCTH exploits concepts from symbolic dynamics [7] to generate aperiodic spreading sequences that, in contrast to fixed (periodic) PN sequences, depend on the input data. The PCTH scheme combines pseudo-chaotic encoding with a multilevel pulse-position modulation.

In this paper we present a multi-access technique for the PCTH communication scheme, that we call MA-PCTH. In MA-PCTH, each pulse transmitted by the original PCTH scheme is replaced by a pulse train, different for each user. Each pulse train represents the user "signature", very much like in CDMA (code-division multiple access) schemes [8], but now in the time domain. The signal for each user is then demodulated using a pulse-correlator followed by maximum likelihood detection [9].

II. MULTI-ACCESS PSEUDO-CHAOTIC TIME HOPPING

In this section we recall the basics of PCTH [6] and MA-PCTH [11]. PCTH exploits symbolic dynamics to embed user input data into a pseudo-chaotic sequence. A simple example of a chaotic map is the *Bernoulli shift* [10], defined as:

$$x_{k+1} = 2x_k \mod 1 \tag{1}$$

The state x can be expressed as a binary expansion:

$$x = 0.b_1 b_2 b_3 \ldots = \sum_{j=1}^{\infty} 2^{-j} b_j, \tag{2}$$

with b_j equal to either "0" or "1", and $x \in I = [0, 1)$.

In PCTH, the Bernoulli shift (1) is approximated by a finite-length (M-bit) shift register, R. In general, the shift register may be followed by a transformation unit for generating more complex chaotic maps. Then, the signal is converted to an analog voltage by a D/A converter and used to drive a pulse-position modulator (PPM). Each pulse is positioned, dependent on the pseudo-chaotic modulation, within a periodic frame of period T_F. The pulses can occur at any of $N = 2^M$ discrete time instants, where M is the number of bits in the shift register, R. The PCTH receiver comprises a pulse correlator, matched to the pulse shape, followed by a pulse-position demodulator (PPD) and a detector. In the simplest case, the binary message may be retrieved by means of a threshold detector at the output of the PPD. For more details, see reference [11].

Fig. 1 shows a simplified block diagram (for the generic j-th user) of the MA-PCTH multi-access scheme. The input to the system is an i.i.d. (independent identically distributed) source of binary data, $b_k^{(j)}$, where the subscript

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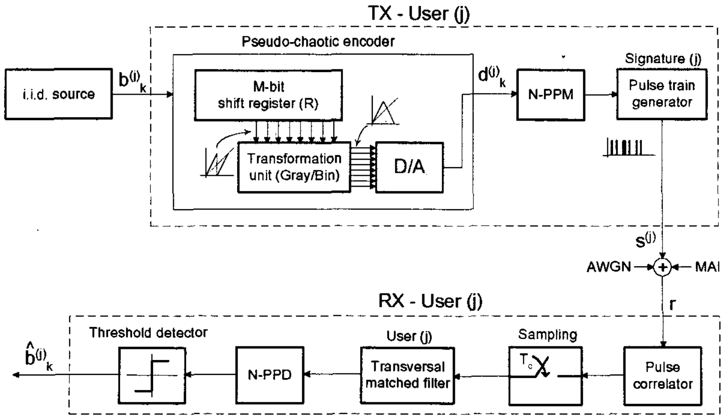


Fig. 1. Simplified block diagram of the MA-PCTH scheme.

denotes the k-th bit. The input sequence feeds the pseudo-chaotic encoder whose output, $d_k^{(j)}$, drives the N-PPM modulator producing the time hopping. In MA-PCTH, the output of the modulator is used to trigger a pulse train generator corresponding to the specific signature, $c^{(j)}$, associated with the j-th user. In this work, we consider a slotted system where the periodic frames (of period T_F) corresponding to the different users are synchronized with each other. Each frame is sub-divided into N slots of duration $T_s (= T_F/N)$. In turn, each slot contains N_c chips; correspondingly, the chip time is given by $T_c = T_s/N_c$. In our analysis we assume that, for each user, the pulse trains are confined within the slot time T_s, *i.e.* the user signatures do not invade adjacent slots. This also implies that, within a given frame time T_F, two generic users (j) and (h) will either transmit in different slots or collide.

The transmitted signal, $s^{(j)}(t)$, for the j-th user can be expressed, for each frame, as:

$$s^{(j)}(t) = \sum_{l=0}^{N_c-1} c_l^{(j)} w_p(t - lT_c - d_k^{(j)} T_s), \qquad t \in [0, T_F]$$

where $c_l^{(j)} \in \{0,1\}$ $(l = 0, \ldots, N_c - 1)$ is the binary sequence representing the j-th user's signature. $w_p(t)$ is the pulse waveform that in this work is assumed to be rectangular:

$$w_p(t) = \begin{cases} 1, & 0 < t < t_p \\ 0, & otherwise \end{cases}$$

where t_p is the pulse duration, and $t_p < T_c$. So, for each information bit, $b_k^{(j)}$, a pseudo-chaotic iterate $d_k^{(j)} \in \{0, \ldots N-1\}$ is generated and the pulse train for the j-th user is transmitted in the corresponding slot, within the frame.

In general, with N_u users transmitting simultaneously, the input to the j-th receiver will be: $r(t) = s^{(j)}(t) + n(t) + n_u^{(j)}(t)$, where the term $n_u^{(j)}(t)$ accounts for the multi-access interference (MAI) caused by the remaining $(N_u - 1)$ users.

Referring to Fig. 1, the j-th receiver comprises a pulse correlator for the pulse waveform $w_p(t)$. The output of the correlator, for slot s, is given by:

$$\rho_{si} = \int_{iT_c+sT_s}^{(i+1)T_c+sT_s} w_p(\tau) r(\tau) d\tau, \qquad i = 0, \ldots, N_c - 1$$

which is sampled at each chip time, T_c. The samples ρ_{si} are then fed into a (digital) transversal matched filter [12]. In the case under consideration the weights, a_i, should coincide with the user signature, that is: $a_i \equiv c_i^{(j)}, \quad i = 0, \ldots, N_c - 1$. Thus, the output of the transversal filter is, for slot s:

$$y_s^{(j)} = \sum_{i=0}^{N_c-1} c_i^{(j)} \rho_{si}, \qquad s = 0, \ldots, N-1 \tag{3}$$

where the subscript, s, runs over the number of slots per frame. The pulse-position demodulation is carried out by applying a maximum-likelihood criterion on each frame. Namely, the most likely slot, $\hat{s}^{(j)}$, is: $\hat{s}^{(j)} = \arg\max_s \{y_s^{(j)}, s = 0, \ldots, N-1\}$. Finally, the estimate $\hat{b}_k^{(j)}$ of the transmitted bit (for the j-th user) can be obtained by means of a threshold detector.

III. THEORETICAL BIT ERROR RATE

We now turn to analyze the bit-error-rate performance of the MA-PCTH scheme for an arbitrary number, N_u, of users. Without loss of generality, we will consider the BER of user 1 in the presence of AWGN and MAI introduced by the $(N_u - 1)$ other users. The cross-correlation value with each user is normalized to the auto-correlation value of user 1. Let's consider the following error event denoted by D. There are n slots indexed by $i = 1, \ldots, n$, different from the slot used by user 1, and slot i contains α_i interference terms. The slot occupied by user 1 receives contributions from $N_u - 1 - \sum_{i=1}^{n} \alpha_i$ interferers and all the others slots are not used. The probability that user 1 is detected in the wrong slot is given by [11]:

$$Pr(error|D) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tag{4}$$

$$\left[1 - \prod_{i=1}^{n} \Phi\left(y - \sqrt{2S_0}\sum_{j=1}^{\alpha_i}\gamma_{ij}\right)\Phi(y)^{N-1-n}\right] \cdot$$

$$\cdot \exp\left[-\frac{1}{2}\left(y - \sqrt{2S_0}\left(1 + \sum_{j=1}^{N_u-1-\sum_{i=1}^{n}\alpha_i}\gamma_j\right)\right)^2\right] dy$$

where γ_{ij} (resp. γ_j) represents the cross-correlation between user 1 and interferer j in slot i (resp. the slot occupied by user 1). The average probability of error is:

$$P_e = \frac{1}{N^{N_u - 1}}\left[Pr(A)Pr(error|A) \right.$$
$$\left. + Pr(B)Pr(error|B) + Pr(C)\right] \tag{5}$$

A denotes the event where all users transmit in the same slot. B is the event where all users transmit in different slots and the combination events, denoted by C, are all the other possible interference events. In order to calculate the average probability of error we need an expression for the probability of each the possible interference events $P(A)$, $P(B)$, etc. For the sake of simplicity, we will assume the set of user signatures have equal cross-correlation to every other user. $Pr(C)$ is the weighted average of the probability of error of all these combination events:

$$Pr(C) = \sum_{\lambda=1}^{\lambda_{max}} \sum_{a_\lambda=2}^{a_{\lambda-1}} \cdots \sum_{a_3=2}^{a_2} \sum_{a_2=2}^{a_1} \sum_{a_1=2}^{a_0} \tag{6}$$

$$\binom{N_u}{a_1}\binom{N_u - a_1}{a_2}\cdots\binom{N_u - a_1 - \ldots - a_{\lambda-1}}{a_\lambda} \cdot$$

$$(N-1)(N-2)\ldots(N - (N_u - \sum_{i=1}^{\lambda-1} a_i) - (\lambda - 1)) \cdot$$

$$\left(\frac{1}{\beta_2!\beta_3!\ldots}\right)\frac{1}{N_u}\left[a_1 Pr_e(A_1^1, A_2, \ldots, \alpha_0) + \right.$$

$$\left. a_2 Pr_e(A_1, A_2^1, \ldots, \beta_0) + \ldots + \alpha_0 Pr_e(A_1, A_2, \ldots, \alpha_0^1)\right]$$

where, $a_0 = min(N_u - 1, N_u - \sum_{i=2}^{\lambda} a_i)$. β_2 is the number of slots in which 2 users transmitted, and β_3 is the number of slots in which three users transmitted, etc.

In Eq. (6), $\lambda_{max} = \lfloor(\frac{N_u}{2})\rfloor$ is the maximum number of different possible interference events within a single frame. This occurs when $\frac{N_u}{2}$ pairs of users interfere. The number of users who transmit in slots with no interfering users is $\alpha_0 = N_u - \sum_{i=1}^{\lambda} a_i$. In the above expression, the events $A_1, \ldots, A_\lambda$ correspond to a_1 users transmitting in the same slot, a_2 users transmitting in the same slot but different than the A_1 users, etc. The superscript 1 indicates that user 1, the user of interest, is included in that set.

IV. ERROR FLOOR CONDITION

If only two users are present, for each frame, they transmit either in the same slot or in different slots. The maximum interference, due to the total cross-correlation, γ_T, has just a single term, that due to user 2. If, as required to discriminate among different users, the cross-correlation between users 1 and 2 is less than unity no error floor is present. The condition for the existence of an error floor is:

$$\gamma_T = \sum_{k=2}^{N_u} \gamma_k \geq 1 \tag{7}$$

where γ_k is the cross-correlation between user 1 and user k.

In general, if three or more users are present, the value of the error floor is the sum of the event probabilities, $Pr(C)$, where the condition (7) is met. From Eq. (5) the error floor can be expressed as:

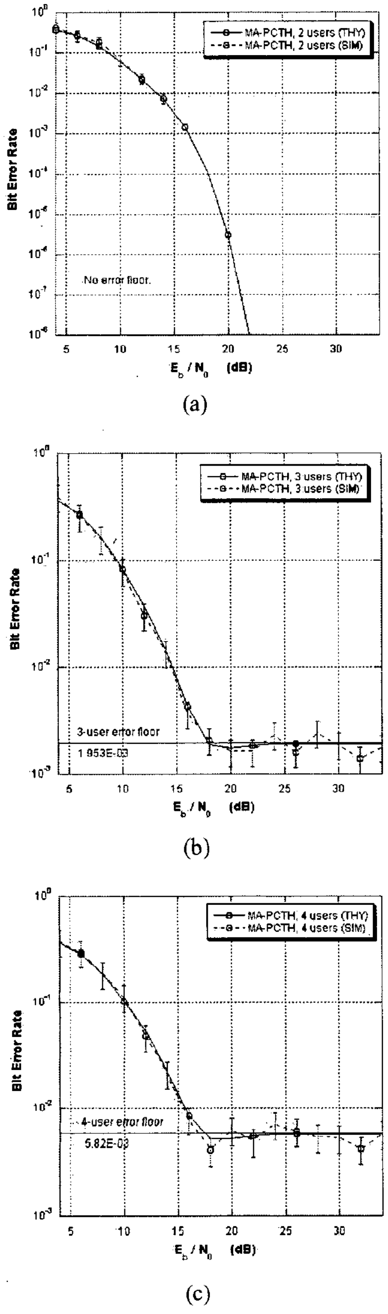
$$Pr(error)_{floor} = \left.\frac{1}{N^{N_u-1}} Pr(C)\right|_{SNR \to \infty} \tag{8}$$

V. SIMULATION RESULTS

This section reports the simulation results for the MA-PCTH scheme and compares them with the theoretical predictions. The results of our analysis are presented in terms of BER versus the ratio E_b/N_0, where E_b is the energy per user bit and N_0 is the single-sided spectral noise power density $(\sigma_n^2 = N_0/2)$ of the AWGN.

In our previous work we have shown that the BER performance improves with decreasing cross-correlation [11]. This is consistent with the fact that orthogonal signaling results in the best possible BER performance.

In the simulations we used $M = 8$ bits corresponding to $N = 256$ PPM levels, with $N_c = 32$ chips/slot. In each of the multi-user cases a 32-bit signature sequence was assigned to the different users. The binary sequences that we chose to use were randomly selected. One constraint imposed on the sequence selection process was that each sequence should contain an equal number of ones (specifically



(a)

(b)

(c)

Fig. 2. Simulated and analytical BER performance of the two-, three-, and 4-user cases of the MA-PCTH scheme ($\gamma = 0.5625$).

16 ones and 16 zeros). This maintains a constant energy across all users. The randomly selected sequences have a periodic cross-correlation value to user 1, the user of interest, of 0.5625. In other words, we chose: $\gamma_k = \gamma = 0.5625$, $\forall k$. Fig. 2 shows the analytically calculated (using Eq. (4)) and simulated two-, three-, and four-user bit error rate curves. Since the maximum cross-correlation in the two-user case does not meet the criterion of Eq. (7), no error floor is present (see Fig. 2(a)). Actually, Eq. (7) is first satisfied for three users. The maximum cross-correlation, γ_T in the three-user case is $2\gamma = 1.125$. The three-user case is shown in Fig. 2(b) The value of the error floor is the corresponding coefficient, $P(C_2) = (N-1)/N^2$. Converting from symbol error rate to BER using, $BER(C_{23}) = \frac{2^{M-1}}{2^M-1}P(C_{23})$, we find the floor to be 1.953E-03. Eq. (6) can be used to calculate the expected performance of the 4-user case. In Fig. 2(c) we show the corresponding simulated and calculated BER performance. Notice that the error-floor value increases with the number of users.

VI. CONCLUSIONS

The success of MA-PCTH as a communication system depends on how many users can be supported at a sufficiently low error rate and a sufficiently high data rate. In this paper we have presented a general expression for the average probability of error for synchronous MA-PCTH. We have shown that the BER is dependent on the total cross-correlation and thus the number of users. Also, an error floor exists if the total cross-correlation exceeds one.

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