A MULTI-ACCESS SCHEME FOR UWB RADIO USING PSEUDO-CHAOTIC TIME HOPPING

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ABSTRACT

Pseudo-chaotic time hopping (PCTH) is a recently proposed encoding/modulation scheme for UWB (ultra-wide band) impulse radio. PCTH exploits concepts from symbolic dynamics to generate aperiodic spreading sequences. In this paper we present a multiple access technique suitable for the pseudo-chaotic time hopping scheme.

1. INTRODUCTION

Over the last decade there has been a great interest in UWB impulse radio communication systems. These systems make use of ultra-short duration ($<$ 1ns) pulses which yield ultra-wide bandwidth signals characterized by extremely low power spectral densities [1, 2]. UWB systems are very promising for short-range wireless communications as they potentially combine reduced complexity with low power consumption, low probability of intercept (LPI) and immunity to multipath fading. The successful deployment of the UWB technology depends strongly on the development of efficient multi-access techniques. Existing UWB communication systems employ pseudo-random noise (PN) time hopping for multiple access purposes combined with pulse-position modulation (PPM) for encoding the digital information. An analysis of the multi-user capabilities of such systems has been presented by Scholtz *et al.* in [3, 4, 5].

Recently it has been suggested to use aperiodic (chaotic) codes in order to enhance the spread-spectrum characteristics of UWB systems by removing the spectral features of the signal transmitted, thus resulting in a low probability of intercept. In this work we consider a recently proposed modulation scheme for UWB impulse radio, called pseudo-chaotic time hopping (PCTH) [6]. PCTH exploits concepts from symbolic dynamics [7] to generate aperiodic spreading sequences that, in contrast to fixed (periodic) PN sequences, depend on the input data. The PCTH scheme combines pseudo-chaotic encoding with a multilevel pulse-position modulation. The pseudo-chaotic encoder operates on the input data in a way that resembles a convolutional code [8]. Its output is then used to generate the time hopping sequence resulting in a random distribution of the inter-pulse intervals, thus in a noise-like spectrum.

In this paper we present a multiple access technique for the PCTH communication scheme, that we call MA-PCTH. The basic idea consists of replacing each pulse transmitted by the original PCTH scheme with a pulse train, different for each user. The latter represents the user "signature" very much like in CDMA (code-division multiple access) schemes [9], but in the time domain. Each user is demodulated with a filter matched to its signature.

2. PSEUDO-CHAOTIC TIME HOPPING

In this section we recall the basics of the single-user PCTH scheme [6]. To this aim we start by recalling some useful concepts about the shift map and its symbolic dynamics. Symbolic dynamics may be defined as a "coarse-grained" description of the evolution of a dynamical system [7]. The idea is to partition the state space and to associate a symbol to each partition. Then, a trajectory of the dynamical system can be analyzed as a symbolic sequence. A simple instance of a chaotic map is the *Bernoulli shift* [10], defined as:

$$x_{k+1} = 2x_k \mod 1 \tag{1}$$

The state x can be expressed as a binary expansion:

$$x = 0.b_1b_2b_3 \ldots = \sum_{j=1}^{\infty} 2^{-j} b_j, \tag{2}$$

with b_j equal to either a "0" or a "1", and $x \in I = [0, 1]$. For this map a Markov partition [7] can be selected by splitting the interval $I = [0, 1]$ in the two subintervals $I_0 = [0, 0.5)$ and $I_1 = [0.5, 1]$. Then, in order to obtain a symbolic description of the dynamics the binary symbol "0" and "1" are associated with the subintervals I_0 and I_1, respectively.

In PCTH the Bernoulli shift (1) is approximated by a finite-length (M-bit) shift register R. From the viewpoint of information theory the shift register implementing the

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Bernoulli shift may be seen as a form of convolutional coding [7]. The memory of the structure is given by the shift register which stores the last M input bits. Each input bit causes an output of M bits; thus the overall rate is $1/M$. In general, the shift register may be followed by a transformation unit for generating more complex chaotic maps.

In the PCTH scheme, the output of the pseudo-chaotic encoder is used to drive a pulse position modulator. Namely, each pulse is allocated, according to the pseudo-chaotic modulation, within a periodic frame of period T_F. In other words, only one pulse is transmitted within each frame time. If the pulse occurs in the first half of the frame a "0" is being transmitted, otherwise a "1". Each pulse can occur at any of $N = 2^M$ discrete time instants, where M is the number of bits in the shift register R.

The PCTH receiver comprises a pulse correlator, matched to the pulse shape, followed by a pulse position demodulator (PPD) and a detector. In the simplest case the binary information may be retrieved by means of a threshold discriminator at the output of the PPD.

3. MULTIPLE ACCESS FOR PCTH

Fig. 1 shows a simplified block diagram for the proposed multiple access scheme based on pseudo-chaotic time hopping, that we denote by MA-PCTH. The transmitter/receiver architecture shown Fig. 1 refers to the generic j-th user. The input to the system is an i.i.d. (independent identically distributed) source of binary data, $b_k^{(j)}$, where the lower index denotes the k-th bit. The input sequence feeds the pseudo-chaotic encoder, whose operation has been described in Sec. 2. As in PCTH, the output of the pseudo-chaotic encoder, $d_k^{(j)}$, drives the N-PPM modulator thus producing the time hopping. In MA-PCTH, though, the output of the modulator is used to trigger a pulse train generator corresponding to the specific signature, $c^{(j)}$, associated with the j-th user. In this work, for simplicity, we consider a slotted system where the periodic frames (of period T_F) corresponding to the different users are synchronized with each other. Each frame is sub-divided into N slots of duration $T_s (= T_F/N)$. In turn, each slot contains N_c chips; correspondingly the chip time is given by $T_c = T_s/N_c$. In our analysis we assume that, for each user, the pulse trains are confined within the slot time T_s, *i.e.* the user signatures do not invade adjacent slots. This also implies that, within a given frame time T_F, two generic users (j) and (k) will either transmit in different slots or collide. The situation, for a single frame period, is illustrated in Fig. 2.

The transmitted signal, $s^{(j)}(t)$ for the j-th user can be expressed, for each frame, as:

$$s^{(j)}(t) = \sum_{l=0}^{N_c-1} c_l^{(j)} w_p(t - lT_c - d_k^{(j)} T_s), \qquad t \in [0, T_F]$$

where $c_l^{(j)} \in \{0, 1\}$ ($l = 0, \ldots, N_c - 1$) is the binary sequence representing the j-th user signature. On the other hand, $w_p(t)$ is the pulse waveform that in this work is assumed to be rectangular:

$$w_p(t) = \begin{cases} 1, & 0 < t < t_p \\ 0, & otherwise \end{cases}$$

where t_p is the pulse duration, and $t_p < T_c$. So, for each information bit, $b_k^{(j)}$, a pseudo-chaotic iterate $d_k^{(j)} \in \{0, \ldots, N-1\}$ is generated and the pulse train for the j-th user is transmitted in the corresponding slot, within the frame time.

In the case of a single user, the signal received is simply: $r(t) = s(t) + n(t)$, where $n(t)$ is the term due to additive white Gaussian noise (AWGN). In general, with N_u users transmitting simultaneously, the input to the j-th receiver will be:

$$r(t) = s^{(j)}(t) + n(t) + n_u^{(j)}(t)$$

where the term $n_u^{(j)}(t)$ takes into account the multiple-access interference (MAI) caused by the remaining $(N_u - 1)$ users sharing the channel.

Referring to Fig. 1, the j-th receiver comprises a pulse correlator for the pulse waveform $w_p(t)$. The output of the correlator for the s-th slot is given by:

$$\rho_{si} = \int_{iT_c + sT_s}^{(i+1)T_c + sT_s} w_p(\tau) r(\tau) d\tau, \qquad i = 0, \ldots, N_c - 1$$

which is sampled according to the chip time T_c. The samples ρ_{si} are then fed into a (digital) transversal matched filter [11]. In the case under consideration the weights, a_i, should coincide with the user signature, that is:

$$a_i \equiv c_i^{(j)}, \quad i = 0, \ldots, N_c - 1$$

Thus, the output of the transversal filter is, for slot s:

$$y_s^{(j)} = \sum_{i=0}^{N_c-1} c_i^{(j)} \rho_{si}, \qquad s = 0, \ldots, N-1 \tag{3}$$

where the subscript s runs over the number of slots per frame. The pulse-position demodulation is carried out by applying a maximum-likelihood criterion on each frame. Namely, the most likely slot, $\hat{s}^{(j)}$, is:

$$\hat{s}^{(j)} = \arg\max_s \{y_s^{(j)}, s = 0, \ldots, N-1\}$$

that is selecting the largest sample at the output of the transversal filter, for each frame period T_F. Finally, the estimate $\hat{b}_k^{(j)}$ of the transmitted bit (for the j-th user) can be obtained by means of a threshold detector.

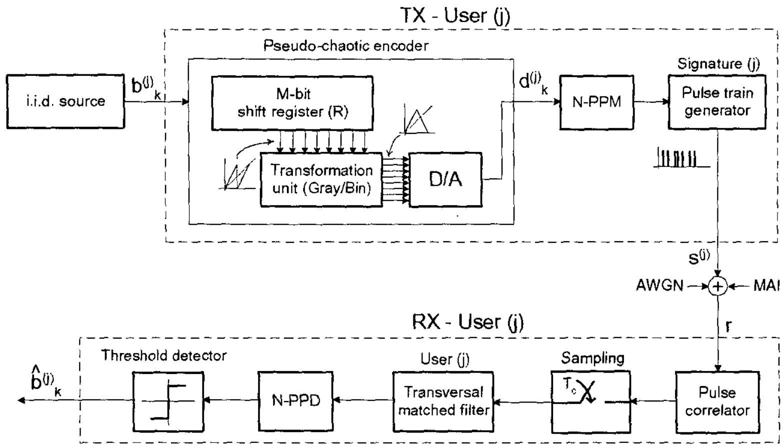

Fig. 1. Simplified block diagram of the MA-PCTH scheme.

4. BER PERFORMANCE: THEORETICAL ANALYSIS

In this section we analyze the BER performance of the MA-PCTH scheme. We assume that the receiver tries to demodulate the data transmitted by user 1 in the presence of multiple access interference introduced by the $N_u - 1$ other users.

4.1. Two-User Case

For each frame, the two users transmit either in a different slot or in the same one. By denoting these mutually exclusive events with A and B, respectively, then the probability of detecting user 1 in the wrong slot, P_e, is given by

$$P_e = Pr(error|A)P(A) + Pr(error|B)P(B),$$

where $P(A) = (N-1)/N$ (resp. $P(B) = 1/N$) denotes the probability of A (resp. B) and $Pr(error|A)$ (resp. $Pr(error|B)$) is the probability of detecting user 1 in the wrong slot given A (resp. given B). $Pr(error|A)$ is obtained by modifying the symbol error probability of N-ary orthogonal signaling [8] by considering that an interference term due to user 2 does not appear in the slot occupied by user 1:

$$P(error|A) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Big[1 - \Big(1 - Q(y - \sqrt{2S_0\gamma^2})\Big) (1 - Q(y))^{M-2}\Big] e^{-\frac{(y-\sqrt{2S_0})^2}{2}} dy, \quad (4)$$

where: $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-\frac{t^2}{2}} dt$, $S_0 = E_1/N_0$ is the signal-to-noise ratio of user 1, with E_1 the transmitted energy of user 1, and $\gamma = \sum_{i=0}^{N_c-1} c_i^{(1)} c_i^{(2)}$ is the periodic cross-correlation between user 1 and 2. In a similar way, $Pr(error|B)$ is obtained by considering that an interference term due to user 2 appears in the slot occupied by user 1:

$$Pr(error|B) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Big[1 - (1 - Q(y))^{M-1}\Big] e^{-\frac{(y-\sqrt{2S_0}(1+\gamma))^2}{2}} dy, \quad (5)$$

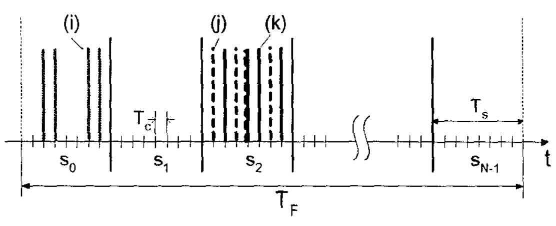

Fig. 2. Sketch of the periodic frame for the MA-PCTH scheme with three users. The frame period, T_F, is divided into N slots, each of duration $T_s (= T_F/N)$. Note the different "signatures" associated with the different users. Users *(j) and (k) exhibit a collision in the third slot* (s_2).

4.2. General case

We generalize the previous considerations to the following error event denoted by C. There are n slots indexed by $i = 1, \ldots, n$ different from the slot used by user 1, and slot i contains α_i interference terms. The slot occupied by user 1 receives contributions from $N_u - 1 - \sum_{i=1}^{n} \alpha_i$ interferers and all the others slots are not used. The probability that user 1 is detected in the wrong slot is given by:

$$Pr(error|C) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty}$$

$$\left[1 - \prod_{i=1}^{n} \left(1 - Q\left(y - \sqrt{2S_0}\sum_{j=1}^{\alpha_i} \gamma_{ij}\right)\right) (1 - Q(y))^{N-1-n}\right] \cdot$$

$$\cdot \exp\left[-\frac{1}{2}\left(y - \sqrt{2S_0}\left(1 + \sum_{j=1}^{N_u - 1 - \sum_{i=1}^{n} \alpha_i} \gamma_j\right)\right)^2\right] dy, \tag{6}$$

where γ_{ij} (resp. γ_j) represents the cross-correlation between user 1 and interferer j in slot i (resp. the slot occupied by user 1).

We now convert the probability of detecting user 1 in a wrong slot P_e to the bit error probability P_b. The errors which consist of confusing the slot used by user 1 with any of the other $N-1$ slots are equiprobable and occur with probability

$$\frac{P_e}{N-1} = \frac{P_e}{2^M - 1}.$$

Let's assume without loss of generality that the binary information digit transmitted by user 1 is zero; then the probability that the receiver makes a bit error is the probability of confusing the slot where user 1 is transmitting with any of the last $N/2$ slots in the frame. Thus,

$$P_b = \frac{2^{M-1}}{2^M - 1} P_e \approx \frac{P_e}{2}, \quad M >> 1. \tag{7}$$

5. SIMULATION RESULTS

This section reports the simulation results for the MA-PCTH scheme and compares them with the theoretical predictions. The results of our analysis are presented in terms of BER probability versus the ratio E_b/N_0, where E_b is the energy per user bit and N_0 is the single-sided spectral noise power density ($\sigma_n^2 = N_0/2$) of the AWGN.

In the simulations we used $M = 8$ bits corresponding to $N = 256$ PPM levels, with $N_c = 32$ chips/slot. In each of the multi-user cases a unique 32-bit signature sequence was assigned to the different users. The binary sequences that we chose to use were randomly selected. The only constraint imposed on the sequence selection process was that each sequence should contain an equal number of ones (specifically 16 ones and 16 zeros). This maintains a constant energy across all users. The randomly selected sequences have periodic cross-correlation values to user 1, the user of interest, of 0.375, 0.4375 and 0.5625, respectively. The family of curves in Fig. 3 shows the two-user simulated performance with each of these cross-correlation values. Note how the BER performance improves with decreasing cross-correlation. As the cross-correlation of the second user's sequence increases, the probability of choosing the slot in which user 2 transmitted increases and so the intended user's probability of error increases. This is consistent with the fact that orthogonal signaling results in the

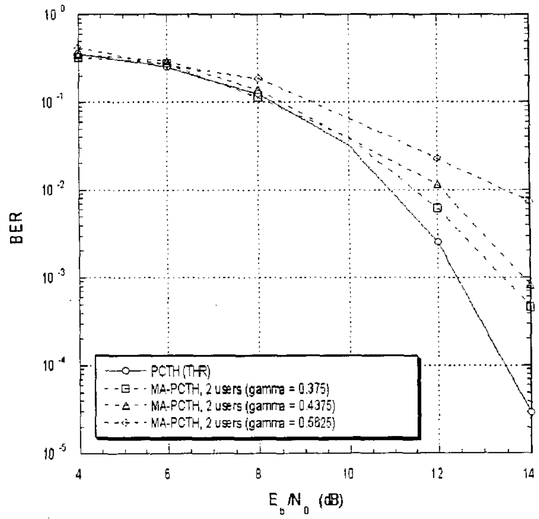

Fig. 3. Simulated BER performance of two-user MA-PCTH scheme for various values of cross-correlation.

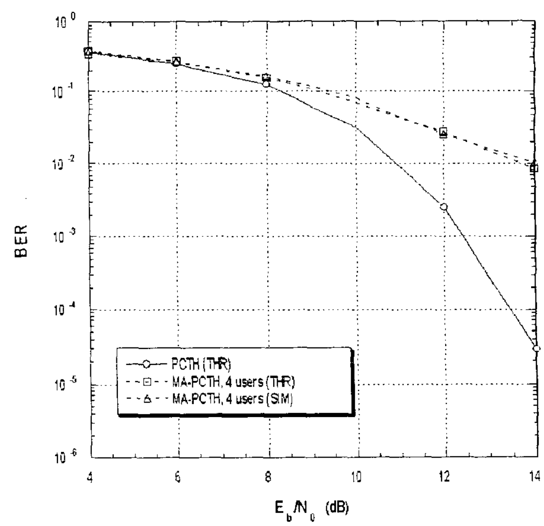

Fig. 4. Simulated vs. analytical BER performance of the four-user MA-PCTH scheme. Users 2,3,4 have cross-correlation values of 0.375, 0.4375, and 0.5625, respectively, with user 1.

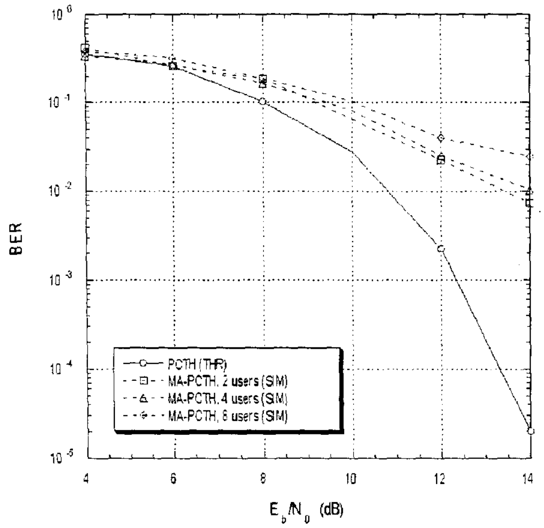

Fig. 5. BER dependence on the number N_u of users.

best possible BER performance.

Fig. 4 shows the analytically calculated, using Eq. (6), and simulated BER performance of four users. The three interfering users had cross-correlation values to the first user of 0.375, 0.4375 and 0.5625, respectively.

Finally, Fig. 5 shows the BER performance of the simulated system as a function of the number of users, N_u. In the simulated two-user data, the sequence with cross-correlation of 0.5625 was used. As the number of users increases, the BER performance is degraded. This is true as long as the cross-correlation increases with an increasing number of users, like in our case.

6. CONCLUSIONS

The success of MA-PCTH as a communication system depends on how many users can be supported at a sufficiently low error rate and a sufficiently high data rate. In this paper we have shown that the BER performance of single-user PCTH is the same as N-ary orthogonal signaling (e.g. N-PPM). For multi-user communications, there are many parameters that need to be optimized in order to produce the best overall system performance. These include the length of the signature sequences, the cross-correlation between sequences, the dimensionality of the pulse-position-modulator and the system data rate capacity. In this work, we have investigated the influence of the periodic cross-correlation between the generic user and each of the other users on the BER, in a synchronous system. Our analysis indicates that the highest cross-correlation amongst interferers tends to dominate the BER performance. For future work, in order to improve the BER performance, one needs to find sets of signature sequences exhibiting constant cross-correlation between any two sequences in the set or that are bounded by an acceptable level. Another interesting variation of the proposed scheme is the non-slotted MA-PCTH case, where $N_c T_c > T_s$. In this case, the length of each user's sequence extends through one slot into an adjacent slot causing intra-symbol interference. When users interfere with each other, they no longer do it in a unique way because there is more than one way that the sequences can overlap. Aperiodic cross-correlations must now be considered. The possible benefits are a potentially improved data rate, simpler implementation and the potential to support more users albeit with a possibly higher BER.

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