PHASE NOISE EFFECTS ON SIGNAL DETECTION
FOR UWB/WIMAX COEXISTENCE

Sanghoon Park, Lawrence E. Larson, and Laurence B. Milstein
Center for Wireless Communication, Dept. of Electrical and Computer Engineering
University of California - San Diego, La Jolla, CA 92037

ABSTRACT

The effect of local oscillator (LO) phase noise of an ultra-wide band (UWB) receiver on the detection of a nearby WiMAX receiver is investigated. An expression for the spectrum broadening effect due to the phase noise is derived, and a practical method is presented to estimate the in-band interference due to the phase noise interaction with an interfering adjacent channel signal.

INTRODUCTION

After the Federal Communications Commission (FCC) authorized the unlicensed use of the frequency band between 3.1 to 10.6 GHz for UWB communication systems [1], interference problems became a concern. One of the systems that share the frequency spectrum with UWB is the worldwide interoperability for microwave access (WiMAX) system. Based on the IEEE 802.16 standard, WiMAX is currently utilizing several frequency bands below 11 GHz, such as the unlicensed 5 GHz band, the licensed 3.5 GHz band, and the licensed 2.5 GHz band [2]. Since the WiMAX system is expected to share the frequency spectrum with UWB, an active WiMAX device detection scheme is highly desirable for UWB devices.

Detecting WiMAX downlink signal is a possible approach, but it has the drawback that it will confine the available frequency spectrum for the UWB device even when no WiMAX receiver is present. As shown in Figure 1, the radius of the UWB system is confined to about 10 m, and so detecting the LO leakage signal from a nearby WiMAX mobile terminal receiver could be used by a UWB device to detect the presence of that WiMAX receiver [3]. However, the detection sensitivity is of major concern due to the very low level of the incoming leakage signal power. In particular, the phase noise of the UWB detector’s RF oscillator can degrade the overall detection sensitivity due to the spectrum broadening effect.

In this paper, we evaluate the effect of receiver phase noise on the in-band interference in an additive white Gaussian noise (AWGN) environment and apply the result to the detection of a WiMAX mobile terminal (MT) by a

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When a WiMAX MT is in the receive mode, a small portion of the receiver LO signal power leaks to the antenna and radiates as shown in Figure 1. Typically, a leakage signal power level of –70 to –90 dBm is expected at the antenna port [4]. This signal is received at the UWB antenna, and is either

\[ r(t) = n_w(t) \]  

or

\[ r(t) = s(t) + n_w(t) \]

where \( s(t) \) and \( n_w(t) \) denote the received leakage signal from the WiMAX MT and the received noise, respectively.

We assume that a UWB device relies on a Discrete Fourier Transform (DFT) to discriminate the unmodulated leakage signal from the background noise. The UWB device determines that \( s(t) \) is absent if the magnitude
response of all the DFT frequency bins fall below a specified threshold. If at least one of the DFT outputs exceeds the threshold, the receiver determines that \( s(t) \) is present. The detection performance can be described by the probabilities of false alarm (\( P_{FA} \)) and missed detection (\( P_{MD} \)).

**SPECTRUM BROADENING EFFECT DUE TO PHASE NOISE**

A simplified block diagram of a direct conversion type receiver is shown in Figure 2. Denoting the phase noise by \( \phi(t) \), and assuming that the standard deviation of \( \phi(t) \) is much less than one radian, the LO signal can be described by

\[
LO(t) = 2\cos(\omega_{LO}t + \phi(t)) \\
= 2\cos(\omega_{LO}t) - 2\phi(t)\sin(\omega_{LO}t). \tag{3}
\]

The frequency response of the front-end bandpass filter (BPF) is shown in Figure 3. It is assumed that the bandwidth of the BPF (\( \omega_B \)) is wide enough to accommodate the WiMAX system. After the BPF, the bandpass noise \( n_l(t) \) is described by

\[
n_l(t) = n_c(t)\cos(\omega_{LO}t + \theta_1) - n_s(t)\sin(\omega_{LO}t + \theta_1) \tag{4}
\]
where \( \theta_1 \), \( n_c(t) \), and \( n_s(t) \) denote an arbitrary phase, the inphase component, and the quadrature component of \( n_l(t) \), respectively. Since \( n_w(t) \) is a zero-mean Gaussian random process, \( n_c(t) \) and \( n_s(t) \) are also both zero-mean Gaussian random processes, and the power spectral density (PSD) of \( n_c(t) \) is given by

\[
S_{n_c}(\omega) = \frac{\eta_0}{2}[P_{aw/2}(\omega + \omega_X) + P_{aw/2}(\omega - \omega_X)] \tag{5}
\]
where \( \eta_0 \) is the single-sided PSD of the AWGN [5], and \( P_a(x) \) is the pulse function defined by

\[
P_a(x) = \begin{cases} 1, & -a \leq x \leq a \\ 0, & \text{otherwise} \end{cases} \tag{6}
\]
As is well known, \( n_s(t) \) has identical statistics to \( n_c(t) \).

Figure 4 shows \( S_{n_c}(\omega) \) or \( S_{n_s}(\omega) \).

Denoting the impulse response of the lowpass filter (LPF) by \( h_L(t) \), the output of the LPF is given by

\[
n_s(t) = [n_A(t) + \phi(t)n_B(t)]*h_L(t) \tag{7}
\]
where

\[
n_A(t) = n_c(t)\cos \theta_1 - n_s(t)\sin \theta_1 \tag{8}
\]
and

\[
n_B(t) = n_c(t)\sin \theta_1 + n_s(t)\cos \theta_1. \tag{9}
\]
We assume that $\omega_{LO}$ is sufficiently large so that the double frequency terms after the multiplier are filtered out by the LPF. Note that $(\phi)$ and $(Bn)$ are also zero-mean Gaussian random processes. Since the autocorrelations of both $(\phi)$ and $(Bn)$ are identical to that of $(\omega)$, the PSD of either $(\phi)$ or $(Bn)$ is given by (5).

In (7), we define $n_b(t)$ as follows:

$$n_b(t) \triangleq \phi(t)n_b(t).$$  \hfill (10)

Since $\phi(t)$ and $n_b(t)$ are independent, the PSD of $n_b(t)$ is given by

$$S_{n_b}(\omega) = S_\phi(\omega)\ast S_{n_b}(\omega).$$  \hfill (11)

In order for the LO signal to achieve a precise frequency, a phase-locked loop (PLL) is widely used. Figure 5 shows the LTI phase-domain model of a typical charge-pump PLL in the locked condition. Since the excess phase of the reference signal is small [6], [7], it is assumed that the phase noise of the LO is dominated by $n_{VCO}(t)$. From Figure 5,

$$\phi(s) = \frac{sK_V}{s^2 + 2\zeta_0\omega_0s + \omega_0^2}n_{VCO}(s)$$  \hfill (12)

where

$$\omega_0 = \sqrt{\frac{K_\phi K_V\omega_0}{M_F}}.$$  \hfill (13)

and

$$\zeta_0 = \frac{1}{2} \frac{K_DK_V}{M_F\omega_0}. \quad (14)$$

Since the quantity $\zeta_0$ is typically close to unity, the double-sided PSD of $\phi(t)$ is given by

$$S_\phi(\omega) \approx \frac{\omega^2K_\phi^2}{(\omega^2 + \omega_0^2)^2} \left(\frac{\eta_{VCO}}{2}\right)$$  \hfill (15)

where $\eta_{VCO}$ and $K_V$ denote the single-sided PSD of $n_{VCO}(t)$ and the frequency gain of the VCO, respectively. It is noted that the phase noise spectrum is often described by the relative power ratio around the fundamental oscillation frequency, $f_{LO}$ [8]. This can be expressed as

$$\mathcal{L}(f) = \frac{S_{LO}(f_{LO} + f)}{P_{LO}/2} \quad (16)$$

where $S_{LO}(f)$ and $P_{LO}$ denote the PSD and power of the LO signal, respectively. For the case of interest [9], [10] $\mathcal{L}(f) = S_\phi(f)$. From (15), at a given frequency offset, $f$, that is greater than $f_0$, $\mathcal{L}(f)$ can be approximated by

$$\mathcal{L}(f) \approx \frac{1}{f^2} \left(\frac{K_\phi^2\eta_{VCO}}{8\pi^2}\right)$$  \hfill (17)

Using (9) and (15) in (11), $S_{n_b}(\omega)$ is given by

$$S_{n_b}(\omega) = \frac{\eta_{VCO}K_\phi^2}{16\pi\omega_0} \left[ V(\omega + \omega_b/2 + \omega_x) 
- V(\omega - \omega_b/2 + \omega_x) + V(\omega + \omega_b/2 - \omega_x) 
- V(\omega - \omega_b/2 - \omega_x) - W(\omega + \omega_b/2 + \omega_x) 
+ W(\omega - \omega_b/2 + \omega_x) - W(\omega + \omega_b/2 - \omega_x) 
+ W(\omega - \omega_b/2 - \omega_x) \right]$$

where

$$V(x) \triangleq \tan^{-1}(x/\omega_0) \quad (19)$$

and

$$W(x) \triangleq \frac{1}{2} \sin[2\tan^{-1}(x/\omega_0)]. \quad (20)$$

Due to the convolution of $S_{n_b}(\omega)$ with $S_\phi(\omega)$, the bandwidth of $n_b(t)$ is wider than that of $n_b(t)$. Therefore, the presence of the phase noise causes the energy of the multiplier input signal to spill over in other signal spectral...
bands. This in-band interference can be a serious problem when a strong out-of-band signal is present.

PHASE NOISE INTERACTION WITH AN ADJACENT CHANNEL SIGNAL

The spectrum broadening effect can introduce in-band interference from an out-of-band signal. Suppose that the UWB detector is searching a particular channel of the WiMAX system, while an adjacent WiMAX channel is concurrently occupied. The received interfering adjacent channel signal is given by

\[ i(t) = i_c(t) \cos \left( \omega_{LO} t + \theta_i \right) \]

\[ -i_s(t) \sin \left( \omega_{LO} t + \theta_i \right) \] \hspace{1cm} (21)

where \( \theta_i \), \( i_c(t) \), \( i_s(t) \), and \( \omega_{ch} \) denote an arbitrary phase, the inphase component, the quadrature component, and the bandwidth, respectively, of the received WiMAX signal. Both \( i_c(t) \) and \( i_s(t) \) can be modeled as independent, identical, WSS bandpass Gaussian processes [11], [12]. Since the OFDM signal consists of a large number of sub-carriers whose spectral width is narrow, the PSD of \( i(t) \) can be approximated as having a rectangular shape [11]-[13]. Thus, the PSD and autocorrelation of \( i_c(t) \) can be approximated as

\[ S_{i_c}(\omega) \approx S_I p_{\omega_{ch}/2}(\omega) \] \hspace{1cm} (22)

and

\[ R_{i_c}(\tau) \approx S_I \sin \left( \omega_{ch} \tau / 2 \right) \pi \tau \] \hspace{1cm} (23)

respectively, where \( S_I \) denotes the single-sided PSD of \( i(t) \), and \( i_s(t) \) has identical statistics to \( i_c(t) \).

The output of the LPF, \( i_3(t) \), can be simplified to

\[ i_3(t) = i_k(t) * h_3(t) \] \hspace{1cm} (24)

where

\[ i_k(t) = i_c(t) \phi(t) \sin (\omega_{ch} t + \theta_i) + i_s(t) \phi(t) \cos (\omega_{ch} t + \theta_i) \] \hspace{1cm} (25)

In arriving (24), we assumed that the LPF provides sufficient attenuation to both the adjacent channel signals and the double frequency terms after the multiplier. Since \( i_c(t) \), \( i_s(t) \), and \( \phi(t) \) are all independent, the autocorrelation function and PSD of \( i_k(t) \) is given by

\[ R_{i_k}(\tau) = R_{i_c}(\tau) R_{i_k}(\tau) \cos \omega_{ch} \tau \] \hspace{1cm} (26)

and

\[ S_{i_k}(\omega) = S_{i_k}(\omega) * S_{i_k}(\omega) * \pi \left[ \delta(\omega - \omega_{ch}) + \delta(\omega + \omega_{ch}) \right] \] \hspace{1cm} (27)

Using (15) and (22), we have

\[ S_{i_k}(\omega) * S_{i_k}(\omega) = \frac{\pi G(f) f^2 S_I}{2 \omega_0} \left[ V(\omega - \omega_{ch}/2) - V(\omega - 3 \omega_{ch}/2) \right. \]

\[ + V(\omega + \omega_{ch}/2) - V(\omega - \omega_{ch}/2) \]

Then, using (17) and (28) in (27),

\[ S_{i_k}(\omega) = \frac{\pi G(f) f^2 S_I}{2 \omega_0} \left[ V(\omega - \omega_{ch}/2) - V(\omega - 3 \omega_{ch}/2) \right. \]

\[ + V(\omega + \omega_{ch}/2) - V(\omega - \omega_{ch}/2) \]

\[ + W(\omega - \omega_{ch}/2) - W(\omega + \omega_{ch}/2) \]

From (24), the autocorrelation function and PSD of \( i_k(t) \) are given by

\[ R_{i_k}(\tau) = R_{i_k}(\tau) * h_3(t) * h_3(-\tau) \] \hspace{1cm} (30)

and

\[ S_{i_k}(\omega) = S_{i_k}(\omega) | H_L(\omega)|^2 \] \hspace{1cm} (31)

respectively, where \( H_L(\omega) \) denotes the Fourier transform of \( h_3(t) \).

Since the leakage signal coming from a nearby WiMAX MT is an unmodulated tone, the bandwidth of the LPF can be much smaller than \( \omega_{ch} \). Therefore, it is assumed that \( S_{i_k}(\omega) \) is almost constant within the bandwidth of the LPF, and so can be approximated as

\[ S_{i_k}(\omega) \approx S_{i_k}(0) | H_L(\omega)|^2 \] \hspace{1cm} (32)

yielding

\[ S_{i_k}(\omega) \approx \frac{\pi G(f) f^2 S_I}{\omega_0} \left[ V(3 \omega_{ch}/2) - V(\omega_{ch}/2) \right. \]

\[ + W(\omega_{ch}/2) - W(3 \omega_{ch}/2) \] \hspace{1cm} (33)

\[ R_{i_k}(\tau) \] can be derived by the inverse Fourier transform of (33). With the ideal brick-wall frequency response of the LPF, \( R_{i_k}(\tau) \) is given by

\[ R_{i_k}(\tau) = \frac{G(f) f^2 S_I \sin (\omega_{ch} \tau)}{\tau \omega_0} \left[ V(3 \omega_{ch}/2) - V(\omega_{ch}/2) \right. \]

\[ + W(\omega_{ch}/2) - W(3 \omega_{ch}/2) \] \hspace{1cm} (34)
where \( \omega_L \) denotes the bandwidth of the LPF. Therefore, the variance of \( i_b(t) \) is given by

\[
\sigma_{i_b}^2 = \frac{\mathcal{S}(f) f^2 S_i \omega_L}{\omega_0} \left[ V\left(3\omega_{ch}/2\right) - V\left(\omega_{ch}/2\right) + W\left(\omega_{ch}/2\right) - W\left(3\omega_{ch}/2\right) \right].
\]  

(35)

The out-of-band signal, \( i(t) \), interacts with the phase noise through the multiplier, and results an in-band interference, \( i_t(t) \).

Denoting the variance of the baseband noise, when there is no phase noise, by \( \sigma_n^2 \), the ratio between the variance of the interference and \( \sigma_n^2 \) is given by

\[
\rho = \frac{\sigma_{i_b}^2}{\sigma_n^2} = \frac{\pi \mathcal{S}(f) f^2 S_i \omega_L}{\eta_0 \omega_0} \left[ V\left(3\omega_{ch}/2\right) - V\left(\omega_{ch}/2\right) + W\left(\omega_{ch}/2\right) - W\left(3\omega_{ch}/2\right) \right].
\]  

(36)

Figure 6 illustrates this ratio with different values of \( S_i/\eta_0 \). As the power of either \( \phi(t) \) or \( i(t) \) increases, the power of \( i_t(t) \) becomes comparable to \( \sigma_n^2 \).

**DETECTION PERFORMANCE DEGRADATION DUE TO PHASE NOISE**

The spectrum broadening effect is now applied to the coexistence problem. When a WiMAX device receives only AWGN, the baseband signal is given by (7). However, since the phase noise of a typical VCO satisfies \( |\phi(t)| \ll 1 \) radian, the second term in the brackets on the right-hand side of (7) is negligible, and so

\[
n_i(t) \approx n_i(t) * h_L(t).
\]  

(37)

Then, \( P_{FA} \) is given by [3]

\[
P_{FA} = 1 - \exp \left( - \frac{S_{TH}^2}{M\sigma_n^2} \right)^M
\]  

(38)

where \( S_{TH} \) denotes the pre-determined decision threshold.

The output of the LPF, when a strong interfering adjacent channel signal is present, should be modified to the sum of (24) and (37), given by

\[
n_i(t) + i_i(t) = [n_i(t) + i_0(t)] * h_L(t).
\]  

(39)

Due to the in-band interference, \( i_i(t) \), in (39), the DFT outputs experience increased variance, and so the magnitude responses are more likely to exceed \( S_{TH} \). The effects of the phase noise on \( P_{FA} \) are shown in Figure 7.

If the leakage signal is present at the input, the detector performance is described by \( P_{MD} \). Without a strong interfering adjacent channel signal, \( P_{MD} \) is given by [3]

\[
P_{MD} = \left[ 1 - Q\left( \sqrt{M \text{SNR}_N}, \frac{2S_{TH}^2}{M\sigma_n^2} \right) \right]^M
\]  

(40)
Figure 8. $P_{MD}$ degradation due to the in-band interference. $P_I$ denotes the power of $i(t)$: $\omega_0 = 2\pi(10^4)$ rad/s, $\omega_L = 2\pi(10^3)$ rad/s, and $\omega_{ch} = 10\pi(10^6)$ rad/s.

where $Q(x,y)$ is the Marcum Q-function, and $SNR_{IN}$ denotes the input signal-to-noise ratio.

When a strong interfering signal is present, the $P_{MD}$ response given by (40) must be affected. Figure 8 shows the $P_{MD}$ responses with and without the phase noise interaction with an adjacent channel signal. It is noted that the $P_{MD}$ curves in Figure 8 exhibit a crossover. At sufficiently high SNR, the DFT response to the leakage signal exceeds $S_{TH}$ so that the in-band interference can only increase $P_{MD}$. However, at sufficiently low SNR, the DFT response to the leakage signal falls below $S_{TH}$, and thus the in-band interference can only lower $P_{MD}$.

CONCLUSION

In this paper, an analysis of the spectrum broadening effect due to the phase noise of a receiver LO signal is presented. It is shown that an adjacent channel signal interacts with the phase noise through a multiplier, and so the spectrum of the multiplier output broadens. As a result, an adjacent channel signal generates strong in-band interference in other channels, and thus degrades the performance. Since the front-end RF BPF cannot provide enough attenuation to the adjacent channel, the local oscillator has to be designed so that the phase noise has been sufficiently suppressed.

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REFERENCE
