

# Highly Linear Bipolar Transconductor For Broadband High-Frequency Applications with Improved Input Voltage Swing

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**Abstract**—An all n-p-n highly linear wide-bandwidth bipolar transconductor ( $G_m$ ) stage is presented based on a variation of Caprio's Quad. The high-frequency (HF) linearity of the improved  $G_m$  cell is examined by a Volterra analysis. The improved  $G_m$  cell has a higher input voltage swing range than other approaches.

## I. INTRODUCTION

Highly linear all n-p-n bipolar transconductors are widely used in many applications to provide faithful voltage-to-current conversion of input stages, such as continuous-time filters [1], mixers [2] and variable gain amplifiers [3]. To meet the required linearity specifications of  $G_m$  cells, various linearity enhancement techniques were developed, such as resistor degeneration [4-5], Caprio's Quad [6], feedback with operational amplifiers [7] and the Cascomp [8].

Because of the cross-coupled translinear configuration, Caprio's Quad has inherent base-emitter voltage ( $V_{BE}$ ) cancellation and provides a very linear voltage-to-current conversion [9]. However, the input voltage swing is also limited due to its cross-coupled base-collector junctions. In light of this, a new bipolar transconductor is proposed based on Caprio's Quad to double the input voltage swing while the high linearity characteristic is maintained. The HF linearity of the improved  $G_m$  cell is investigated by a Volterra series analysis [11] and compared with the resistor degeneration technique and Caprio's Quad.

The paper is organized as follows: Section II describes the improved  $G_m$  cell's operation. Section III investigates its linearity properties. Section IV summarizes the noise performance. Section V calculates the figure-of-merit of each linearity enhancement technique, Conclusions are provided in Section VI.

## II. PRINCIPLE OF OPERATION

The most popular linearization scheme of bipolar differential pairs is with an emitter degeneration resistor [4-5], which is shown in Fig. 1(a). The output current is the well-known

$$\Delta I = \frac{V_{in} + \Delta V_{BE}}{2R_{ee}} \quad (1)$$

where  $\Delta V_{BE}$  represents the base-emitter voltage difference of  $Q_1$  and  $Q_2$ . Equation (1) shows that the imperfect linearity results from the incomplete cancellation of base-emitter voltages of  $Q_1$  and  $Q_2$ , i.e.  $V_{BE1}$  is not exactly equal to  $V_{BE2}$ .

To obtain improved cancellation of base-emitter voltages, Caprio's Quad added one cross-coupled pair  $Q_3$ - $Q_4$  to the original differential pair  $Q_1$ - $Q_2$ , as shown in Fig. 1(b) [6]. The loop voltage can be written as

$$V_{in} - V_{BE1} - V_{BE4} + 2R_{ee}\Delta I + V_{BE3} + V_{BE2} = 0 \quad (2)$$

Because of the cancellation of the base-emitter voltages of  $Q_1$ - $Q_4$ , Caprio's Quad is very linear. Nevertheless, the input voltage swing range of Caprio's Quad is limited due to the cross-coupled base-collector junctions of  $Q_3$  and  $Q_4$ . When  $V_{in}$  exceeds one junction turn-on voltage – roughly 0.8V – the base-collector junction of  $Q_3$  will become forward-biased and high linearity operation will not be preserved.

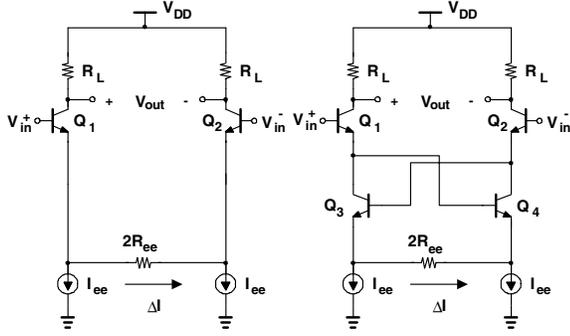
To improve the input voltage swing range, a new  $G_m$  cell is proposed based on a variation of Caprio's Quad and shown in Fig. 1(c). The improved  $G_m$  cell is composed of two translinear loops:  $Q_1$ - $Q_3$ - $Q_4$ - $Q_2$  and  $Q_7$ - $Q_5$ - $Q_6$ - $Q_8$ . The input voltage loop can be expressed as:

$$V_{in} - V_{BE1} - V_{BE3} - 2R_{ee}\Delta I + V_{BE4} + V_{BE2} = 0 \quad (3)$$

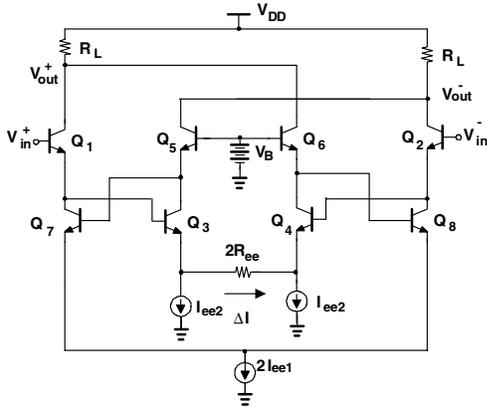
With the aid of the other translinear loop  $Q_7$ - $Q_5$ - $Q_6$ - $Q_8$ , we have

$$V_{BE7} + V_{BE5} - V_{BE6} - V_{BE8} = 0 \quad (4)$$

Because of the cancellation of base-emitter voltages, the improved  $G_m$  cell preserves the highly linearity characteristics of Caprio's Quad. Moreover, the input voltage swing range is roughly doubled due to the aid of the second translinear loop. The base-collector junctions of  $Q_7$ - $Q_3$  and  $Q_4$ - $Q_8$  will not become forward-biased until  $V_{in}$  exceeds two  $V_{BE}$ 's – roughly 1.6V. Hence the input voltage swing range is doubled compared with Caprio's Quad.



(a) (b)



(c)

Fig.1 Different linearity enhancement techniques: (a) differential pair with emitter degeneration resistor (b) Caprio's Quad and (c) improved  $G_m$  cell.

### III. LINEARITY

In communication applications, the linearity is usually characterized by intermodulation distortion ( $IM$ ) and third-order input referred intercept point ( $IIP3$ ) [10]. These weakly nonlinear HF characteristics can be calculated by the Volterra series method [11]. To simplify the Volterra analysis, the following assumptions are made: 1) Current gain is assumed very large, 2) Base-emitter capacitor  $C_{be}$  is estimated as  $g_m$ , where  $g_m$  is the transconductance and  $\tau_f$  is the forward transit time of bipolar transistors [12], 3) The product  $g_m R_{ee}$  is much greater than unity, 4) The ratio  $\Delta f/f$  is much less than unity, where  $f$  is the input signal frequency and  $\Delta f$  is the frequency spacing of the two-tone signal, 5) The source impedance  $R_s$  is purely resistive (as it would be for broadband applications).

#### A. Linearity of Differential Pair with Degeneration Resistor

Suppose a two-tone signal with frequency components  $\Delta f + f$  and  $f$  is fed into differential pair with degeneration resistor, as shown in Fig. 1(a), the third-order input referred intercept point voltage ( $V_{IIP3}$ ) at frequency  $f + 2\Delta f$  is [5]

$$V_{IIP3_{Deg}} \approx 4V_T \left(1 + \frac{I_T R_{ee}}{2V_T}\right)^{\frac{3}{2}} \quad (5)$$

where  $V_T$  is the thermal voltage,  $R_{ee}$  is the emitter degeneration resistor and  $I_T = 2I_{ee}$  is the overall current consumption of the differential pair. Equation (5) shows the well-known result that the  $V_{IIP3_{Deg}}$  relies on the product  $I_T R_{ee}$ .

#### B. Linearity of Caprio's Quad

Through a Volterra series analysis, the third-order intermodulation components ( $IM3$ ) of the output voltage  $V_{out}$  at frequency  $f + 2\Delta f$  can be expressed as:

$$IM3_{cap} \approx \left| \frac{A_{in}^2}{8g_m^3 R_{ee}^3 V_T^2} \frac{f}{f_T} \right| \quad (6)$$

where  $A_{in}$  is the input amplitude (in Volt),  $g_m$  is the transconductance of the transistors, and  $R_{ee}$  is the emitter degeneration resistor in Fig. 1(b). The quantity  $f_T$  is the cut-off frequency of the bipolar transistor, and is evaluated as  $g_m/2\pi C_{be}$ .  $IIP3$  can be solved from (6) by setting  $IM3_{cap} = 1$  as follows:

$$V_{IIP3_{cap}} \approx 2\sqrt{2}V_T \sqrt{\frac{g_m^3 R_{ee}^3 f_T}{f}} \quad (7)$$

where  $V_{IIP3_{cap}}$  represents the third-order input referred intercept point voltage of Caprio's Quad.  $V_{IIP3_{cap}}$  can be further simplified as

$$V_{IIP3_{cap}} \approx \sqrt{\frac{f_T I_T^3 R_{ee}^3}{f V_T}} \quad (8)$$

where  $I_T$  is the overall current consumption. Compared with the differential pair with degeneration resistance,  $V_{IIP3_{cap}}$  is not only a function of the product  $I_T R_{ee}$ , but also a function of the frequency ratio  $f/f_T$ . It can be seen that the linearity can be further improved by using high  $f_T$  bipolar transistors.

#### C. Linearity of the Improved $G_m$ Cell

The  $V_{IIP3}$  of the improved  $G_m$  cell can be computed in a similar manner. Through a Volterra series analysis, the third-order intermodulation components ( $IM3$ ) of output voltage  $V_{out}$  can be expressed as

$$IM3_{imp} \approx \left| \frac{A_{in}^2}{32g_m^3 R_{ee}^3 V_T^2} \frac{f(K+1)^2}{f_T K^5} \right| \quad (9)$$

where  $g_m$  is the transconductance of Q1, Q2, Q7 and Q8.  $K = I_{ee1}/I_{ee2}$  is the tail current source ratio in Fig. 1(c). The  $IIP3$  can be solved from (9) by setting  $IM3_{imp} = 1$  as follows:

$$VIIP3_{imp} \approx 2\sqrt{2}V_T \sqrt{\frac{g_m^3 R_{ee}^3 K^5 f_T}{(K+1)^2 f}} \quad (10)$$

where  $VIIP3_{imp}$  represents the third-order input referred intercept point voltage of the improved  $G_m$  cell.  $VIIP3_{imp}$  can be further simplified as

$$VIIP3_{imp} \approx \sqrt{\frac{I_T^3 R_{ee}^3 K^5 f_T}{V_T (K+1)^5 f}} \quad (11)$$

where  $I_T$  is the overall current consumption of the improved  $G_m$  cell. Equation (11) shows that the improved  $G_m$  cell has roughly the same  $VIIP3$  as Caprio's Quad in (8) with a large value of  $K$ . With a high  $f_T$  device,  $VIIP3_{imp}$  can become very large, which is similar to Caprio's Quad.

To validate our Volterra series calculation, the simulated and calculated  $VIIP3$  for Caprio's Quad and the improved  $G_m$  cell with the same overall power consumption are shown in Fig. 2. Simulation results show close agreement with calculation results.

For an ideal transconductor, a flat transconductance response with respect to input voltage is desired. To verify the improved input voltage swing range, the simulated normalized transconductance of the differential pair with emitter degeneration, Caprio's Quad and the improved  $G_m$  cell is shown in Fig. 3. To highlight the transconductance flatness, a 0.1%  $G_m$  compression point is adopted to define the input voltage swing range. It can be seen that the input voltage swing range of the improved  $G_m$  cell is roughly doubled, and is superior to either Caprio's Quad or the classic emitter degeneration transconductance cell.

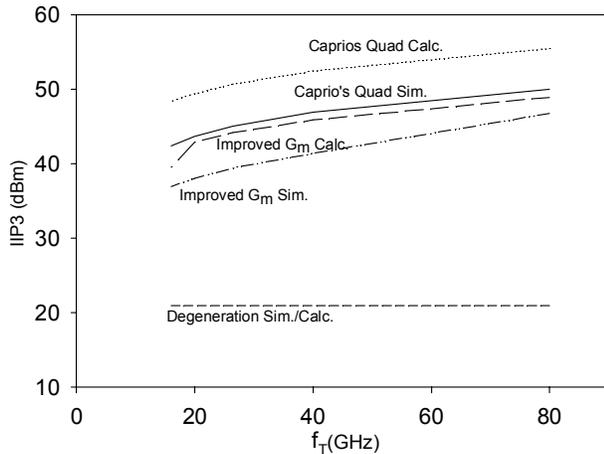


Fig. 2 IIP3 vs. cut-off frequency ( $f_T$ ) of Caprio's Quad, improved  $G_m$  cell and differential pair with emitter degeneration at 20MHz. Simulation parameters :  $R_{ee}=250 \Omega$ ,  $R_L=50\Omega$ ,  $K=2.8$ . Overall current consumption  $I_T=2mA$ .

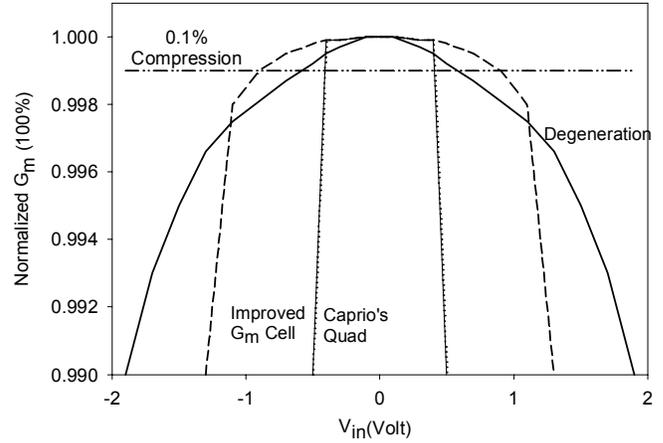


Fig. 3 Simulated normalized  $G_m$  vs. input voltage at 20MHz of Caprio's Quad, improved  $G_m$  cell and differnenail pair with emitter degeneration. The improved  $G_m$  cell has nearly a doubled input dynamic range compared with Caprio's Quad.

#### IV. NOISE PERFORMANCE

The minimum detectable signal of any circuit is usually limited by Noise Factor [13]. The Noise Factor of a differential pair with emitter degeneration resistor, Caprio's Quad and the improved  $G_m$  cell are

$$F_{Deg} = 1 + \frac{1}{2g_m R_s} + \frac{R_{ee}}{R_s} + \frac{1}{R_s R_L} \left( \frac{1 + g_m R_{ee}}{g_m} \right)^2 \quad (12)$$

$$F_{Cap} = 1 + \frac{4g_m R_{ee}^2}{R_s (1 + 2g_m R_{ee})^2} + \frac{R_{ee}}{R_s} + \frac{2R_{ee}^2}{R_L R_s} \quad (13)$$

$$F_{Imp} = 1 + \left( \frac{K+1}{K} \right) \frac{1}{g_m R_s} + \frac{3R_{ee}}{R_s} + 2 \left( \frac{K}{K+1} \right)^2 \frac{R_{ee}^2}{R_s R_L} + \left( 2K + \frac{K}{2(K+1)} \right) \frac{g_m R_{ee}^2}{R_s} \quad (14)$$

where  $F_{Deg}$  is the Noise Factor of the differential pair with emitter degeneration,  $F_{Cap}$  is the Noise Factor of Caprio's Quad, and  $F_{Imp}$  is the Noise Factor of the improved  $G_m$  cell. The base resistance  $r_b$  of bipolar transistors is ignored here to simplify the analysis. It can be seen that  $F_{Imp}$  is slightly higher than  $F_{Cap}$  and  $F_{Deg}$ , as expected.

#### V. FIGURE OF MERIT

A figure-of-merit ( $FOM$ ) for a high-frequency amplifier was defined as follows [14]:

$$FOM_1 = 10 \log \left( \frac{IIP3(mW)}{(F-1)} \frac{1}{P_{dc}(mW)} \right) \quad (15)$$

which normalizes the linearity, Noise Factor  $F$  and power consumption  $P_{dc}$ . However, the above  $FOM$  did not account for the input voltage swing range, which is an important characteristic. Hence, another  $FOM$  is proposed here to compare the transconductors' performance:

$$FOM_2 = 10 \log \left( \frac{IIP3(mW)}{(F-1)} \frac{1}{P_{dc}(mW)} \frac{V_{swing}(Volt)}{V_{DD,min}(Volt)} \right) \quad (16)$$

Where  $V_{swing}$ , defined by the 0.1%  $G_m$  compression point, represents the input swing range for circuit normal operation, and  $V_{DD,min}$  is the minimum required supply voltage. The table of comparison at 20 MHz and 200MHz for emitter degeneration, Caprio's Quad and the improved  $G_m$  cell is shown in Table I. The improved  $G_m$  cell has a similar FOM to the Caprio's Quad but with doubled the input voltage swing range, which is a significant improvement for high-frequency applications.

TABLE I. TABLE OF COMPARISON FOR DIFFERENT TRANSCONDUCTORS

$G_m$ Type	$G_m$ Performance at 20MHz					
	$IIP3$ (dBm)	$NF$ (dB)	$P_{dc}$ (mW)	$V_{swing}$ (Volt)	$V_{DD,min}$ (Volt)	$FOM$ (dB)
Degeneration	20.9	4.9	6.0	0.6	0.9	8.1
Caprio's Quad	50.0	4.4	6.0	0.4	1.6	33.8
Improved $G_m$ cell	46.7	6.5	6.0	0.9	1.6	31.0
$G_m$ Type	$G_m$ Performance at 200MHz					
	$IIP3$ (dBm)	$NF$ (dB)	$P_{dc}$ (mW)	$V_{swing}$ (Volt)	$V_{DD,min}$ (Volt)	$FOM$ (dB)
Degeneration	20.9	4.9	6.0	0.6	0.9	8.1
Caprio's Quad	39.8	4.4	6.0	0.4	1.6	23.6
Improved $G_m$ cell	33.7	6.5	6.0	0.9	1.6	18.0

## VI. CONCLUSION

An all n-p-n bipolar  $G_m$  cell is proposed based on a variation of Caprio's Quad. The HF linearity of the

improved  $G_m$  cell is examined by Volterra analysis. Compared with Caprio's Quad, simulation results show the improved  $G_m$  cell has double the input voltage swing range and a similarly high linearity. The improved  $G_m$  cell is suitable for high-frequency broadband applications because of its high linearity and large input voltage swing range.

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