BER Performance and Spectral Properties of Interleaved Convolutional Time Hopping for UWB Impulse Radio

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***Abstract*—Interleaved convolutional time hopping (ICTH) is a recently proposed coding/modulation scheme for UWB impulse radio. ICTH is based upon a low-rate convolutional code, with optimized distance properties, combined with multilevel pulse-position modulation (PPM). In this context, bit-interleaved coded modulation (BICM) is used to decorrelate the bit errors and, more importantly, to obtain a flat spectrum. In this paper, we present analytical bounds for the BER (bit-error-rate) performance of ICTH and an analysis of its spectral properties.**

I. Introduction

Over the last decade, there has been a great deal of interest in communications based on impulse radio [1], [2], [3], [4]. These systems make use of ultra-short duration pulses which yield ultra-wideband (UWB) signals characterized by low power spectral densities. UWB systems are particularly promising for short-range wireless communications as they potentially combine reduced complexity with low power consumption, low probability-of-intercept (LPI) and immunity to multipath fading. Namely, in [5], it was first demonstrated that UWB signals suffer little from fading and, therefore, only a small fading margin is required to guarantee reliable communications. Existing UWB communication systems employ pseudo-random noise (PN) time hopping combined with modulation schemes like PPM (pulse-position modulation) or PAM (pulse amplitude modulation) for encoding the digital information.

Recently, it has been suggested to use aperiodic (chaotic) codes in order to enhance the spread-spectrum characteristics of UWB systems by removing the spectral features of the transmitted signal, thus resulting in LPI [6]. In particular, the pseudo-chaotic time hopping (PCTH) scheme exploits concepts from symbolic dynamics [7] to generate aperiodic spreading sequences that, in contrast to fixed (periodic) pseudo-noise sequences, depend on the input data. PCTH combines pseudo-chaotic encoding with multilevel pulse-position modulation. The pseudo-chaotic encoder operates on the input data like a convolutional encoder [8]. Its output is then used to generate the time-hopping sequence resulting in a random distribution of the inter-pulse intervals, thus a noise-like spectrum.

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On the other hand, interleaved convolutional time hopping (ICTH) has been first proposed in [9] as an alternative to the pseudo-chaotic time hopping scheme. Basically, in ICTH the shift register present in PCTH is replaced by an equivalent-rate convolutional code with optimized distance properties. The addition of bit-interleaved coded modulation (BICM) has the twofold effect of: i) decorrelating the bit errors for the optimal operation of the Viterbi decoder, and ii) randomizing the output of the convolutional code for spectral whitening purposes. We recall that BICM separates encoding and modulation by means of an interleaver. BICM was first introduced by Zehavi [10] in order to increase the diversity of a coded modulation on the Rayleigh fading channel to the minimum Hamming distance of the code. In [11], Caire *et al.* showed that bit interleaving induces a negligible capacity loss on the AWGN (additive white Gaussian noise) channel. At the receiver, the detected bits are first deinterleaved and then used by the decoder to recover the best estimate of the transmitted information.

In this work we derive analytical bounds for the BER of ICTH and compare its performance to PCTH as well as the signal statistics (specifically the frequency of slot usage) and, more importantly, the spectrum of the transmitted signal.

The paper is organized as follows. In Sec. II we recall the basics of PCTH. Then, in Sec. III we introduce the ICTH scheme. Analytical bounds for both schemes are derived in Sec. IV. Then, in Sec. V we compare the BER bounds vs. the simulation results. Finally, in Sec. VI we present some results illustrating the statistics of the transmitted signal and its spectral properties.

II. Pseudo-Chaotic Time Hopping

In this section, we recall the basics of the PCTH scheme [6] and reinterpret the operation of the Bernoulli shift map utilized in PCTH as a convolutional code.

A. Review of Basics

We start by recalling some useful concepts about the shift map and its symbolic dynamics. Symbolic dynamics may be defined as a "coarse-grained" description of the evolution of a dynamical system [7]. The idea is to partition the state space and to associate a symbol to each partition. Consequently, a

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trajectory of the dynamical system can be analyzed as a symbolic sequence. A simple example of a chaotic map is the *Bernoulli shift* [12], defined as:

$$x_{k+1} = 2x_k \mod 1 \tag{1}$$

The state, x, can be expressed as a binary expansion:

$$x = 0.b_1 b_2 b_3 \ldots = \sum_{j=1}^{\infty} 2^{-j} b_j \tag{2}$$

with b_j equal to either "0" or "1", and $x \in I = [0, 1)$. For this map, a Markov partition [7] can be selected by splitting the interval $I = [0, 1)$ into two subintervals: $I_0 = [0, 0.5)$ and $I_1 = [0.5, 1)$. Then, in order to obtain a symbolic description of the dynamics, the binary symbols "0" and "1" are associated with the subintervals I_0 and I_1, respectively.

In PCTH, the Bernoulli shift (1) is approximated by means of a finite-length (M-bit) shift register, R. Multiplication by 2 in Eq. (1) corresponds to a left shift (b_2 goes to b_1, etc.), while the modulo one operation is realized by discarding the most significant bit (MSB). At each clock impulse the most recent bit of information is assigned the least significant bit (LSB) position in the shift register, while the old MSB is discarded.

In the PCTH scheme, the output of the pseudo-chaotic encoder is used to drive a pulse position modulator. Namely, each pulse is allocated, according to the pseudo-chaotic modulation, within a periodic frame of period T_F. In other words, only one pulse is transmitted within each symbol period, T_F. If the pulse occurs in the first half of the frame a "0" is being transmitted, otherwise a "1". Each pulse can occur at any of $N = 2^{M+1}$ discrete time instants, where M is the number of bits in the shift register. The PCTH receiver comprises a pulse correlator, matched to the pulse shape, followed by a pulse position demodulator (PPD). In the simplest case, the binary information may be retrieved by means of a threshold discriminator at the output of the PPD. In general, though, maximum-likelihood sequence estimation may be performed by using the Viterbi algorithm, as described in [6].

B. Bernoulli Shift Map as a Convolutional Code

From the viewpoint of information theory, the shift register implementing the Bernoulli shift may be seen as a form of convolutional coding [7]. The memory of the structure is given by the shift register which stores the last M input bits. Each input bit causes an output of $(M+1)$ bits; thus the overall rate is $1/(M + 1)$. This is shown schematically (for $M = 7$) in Fig. 1.

In this work, for simplicity, we consider an implementation of the Bernoulli shift map with a 2-bit shift register ($M = 2$) resulting in a rate $1/3$ code. Accordingly, the constraint length of the equivalent convolutional code is $k = (M + 1) = 3$ corresponding to $s = 2^M = 4$ states.

By inspection, one can write down the generator matrix in the standard octal form as:

$$G_{B3} = [4, 2, 1]^T \tag{3}$$

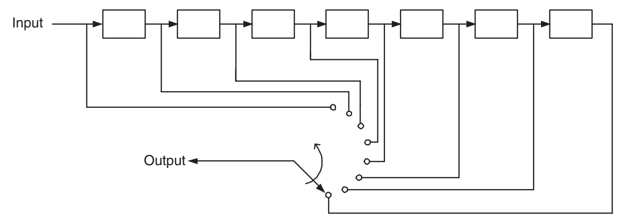

Fig. 1. Equivalent convolutional encoder to the Bernoulli shift map, for $M = 7$ bits.

Using standard methods [8] the transfer function of G_{B3} is found to be,

$$T_{B3}(W, X, Y) = \frac{W^3XY^3}{1 - WXY^3 - W^2XY^3} \tag{4}$$

where the exponent of W is the number of branches traversed, the exponent of X is the Hamming weight of the input bits and the exponent of Y is the Hamming weight of the output bits of the code. This may be verified by analyzing the trellis or state diagram of the code and calculating the ratio of polynomials from the signal flow graph corresponding to path deviating from the zero state and returning to it. Correspondingly, the free distance of the code is $d_{free} = 3$ and there exists one path with this distance.

III. Interleaved Convolutional Time Hopping

Fig. 2 shows a simplified block diagram of the ICTH scheme. Basically, ICTH may be thought as derived from the PCTH scheme by replacing the shift register implementing the Bernoulli shift map with a distance optimized convolutional code of the same rate and the same number of states. In this work, in accordance with the previous section, we consider a rate $1/3$ convolutional code with $s = 4$ states. One such code, found by computer search, has the following generator matrix in octal [8]:

$$G_{I3} = [5, 7, 7]^T \tag{5}$$

The transfer function corresponding to G_{I3} is,

$$T_{I3}(W, X, Y) = \frac{W^3XY^8 - W^4X^2Y^10 + W^4X^2Y^8}{1 - (WXY^2 + W^2XY^2 + W^3X^2Y^2 - W^3X^2Y^4)}.$$

The free distance of T_{I3} is $d_{free} = 8$ and there are three paths with this distance. An implementation of the encoder is easily obtained from the generator matrix. In our ICTH scheme, the convolutional encoder is followed by a bit interleaver whose function is to remove the correlation introduced by the modulation, according to BICM theory [10]. In particular, in [11] it is shown that an interleaving depth, L, equal to 4-5 times the constraint length of the code is sufficient. We emphasize that in the context of ICTH the interleaver has a randomizing effect on the output of the convolutional encoder, thus resulting in a transmitted signal with a flat spectrum (see Sec. VI), a desirable property for UWB impulse radio.

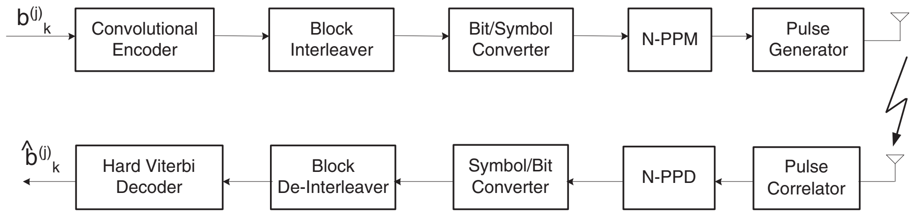

Fig. 2. Simplified block diagram of the ICTH scheme with hard Viterbi decoding. In this work, the bit interleaver is realized by a block interleaver.

IV. BER ANALYTICAL BOUNDS

In this section we derive analytical bounds, in the hard decoding case, both for PCTH and ICTH. Again, for simplicity, we consider a rate $1/n$ convolutional codes, with $n = 3$. The modulation used in each case is 8-PPM, where each symbol transmitted on the channel is mapped to a triplet of encoded bits. The bit interleaving is realized by a block interleaver with interleaving depth equal to $L = 6K$ symbols (that is $6K \times n$ bits), where K is the constraint length of the code. Referring to Fig. 2, note that in the case of hard decoding, the Viterbi decoder is fed directly from the de-interleaver after the most likely transmit symbol has been selected by the pulse-position demodulator (PPD) based on the observed channel energies in each slot.

Without loss in generality, due to the linearity if convolutional codes, we can assume the all-zero sequence was transmitted. Say the decoder is comparing the distance between the received path, l_R, and the all-zero path, l_0, with the distance between l_R and some other path l_D. Let the Hamming distance between l_0 and l_D, $H(0, D) = d$. For d odd, if the Hamming distance between the received sequence and the all-zero sequence $H(0, R) < \frac{1}{2}(d + 1)$, then the received path is closer to the all-zero path than l_D, resulting in the correct path being chosen. Since the all-zero path is being transmitted, the Hamming weight of the received path is also the number of errors experienced over the channel. If $H(0, R) \geq \frac{1}{2}(d + 1)$, then the wrong path will be selected. The probability that the wrong path is selected for d odd in this pairwise comparison is then,

$$P_2(d) = \sum_{k=\frac{d+1}{2}}^{d} \binom{d}{k} p^k (1-p)^{d-k}$$

where p is the probability of bit error over the channel. This is simply the binomially distributed probability that between $\frac{1}{2}(d + 1)$ and d errors occur over the channel. If d is even, the incorrect path is chosen when $H(0, R) > \frac{1}{2}d$. When $H(0, R) = \frac{1}{2}d$, there is a tie between the distance to the all-zero path and the competing path l_D. So, an average of $\frac{1}{2}$ of these cause an error. Then the probability of error for d even is,

$$P_2(d) = \sum_{k=\frac{d}{2}+1}^{d} \binom{d}{k} p^k (1-p)^{d-k} + \frac{1}{2} \binom{d}{\frac{d}{2}} p^{d/2} (1-p)^{d/2}$$

A long information sequence results in many paths that diverge from the all-zero path and remerge at any particular node. The number of such paths depends on the length of the information sequence, causing the probability of error to also be dependent on the length of the information sequence. An upper bound can be expressed as the union bound of the pairwise error probabilities,

$$P_e \leq \sum_{d=d_{free}}^{\infty} a_d P_2(d) \quad (6)$$

where a_d is the number of paths Hamming distance, d, from the all-zero sequence and d_{free} is the free distance of the code. The values of a_d are just the coefficients in the expansion of $T(Y) = T(W, X, Y)|_{W=X=1}$. Note that some values of $a_d = 0$ where there is no term in the expansion of $T(Y) \propto Y^d$. The probability of information bit error can be found by observing that the exponent of X in each term of $T(X, Y) = T(W, X, Y)|_{W=1}$ is equal to the number of ones in the information sequence (and the number of errors from the all-zero information sequence). This is the number of bit errors experienced along the path. By taking the derivative and expanding the result in a Taylor series ($(1 - x)^{-1} = 1 + x + x^2 + x^3 + \ldots$)), we find

$$\left.\frac{\partial T_{B3}(X, Y)}{\partial X}\right|_{X=1} = \beta_{d_{free}} Y^{d_{free}} + \beta_{d_a} Y^{d_a} + \beta_{d_b} Y^{d_b} + \ldots \quad (7)$$

and the probability of bit error is

$$P_b = \sum_{d=d_{free}}^{\infty} \beta_d P_2(d) \quad (8)$$

where again some values of β_d are 0 due to the absence of terms (exponents of X with value d in the expansion of $T(X, Y)$).

A. PCTH

For the 3-bit Bernoulli shift, evaluating Eq. (4) at W=1 results in,

$$T_{B3}(1, X, Y) = T_{B3}(X, Y) = \frac{XY^3}{1 - 2XY^3} \quad (9)$$

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and applying Eq. (7) we find the first three terms of the expansion to be characterized by the following exponents and coefficients

$$[d_{free}, d_a, d_b] = [3, 6, 9]$$

$$[\beta_{dfree}, \beta_a, \beta_b] = [1, 4, 12]$$

resulting in a probability of bit error upper-bounded by,

$$P_b \leq P_2(3) + 4P_2(6) + 12P_2(9) + \ldots \qquad (10)$$

For N-ary orthogonal signals ($N = 2^n$),

$$p = \frac{2^{n-1}}{\sqrt{2\pi}(2^n - 1)} \int_{-\infty}^{\infty} \left[1 - (1 - Q(y))^{N-1}\right] e^{-\frac{\left(y - \sqrt{2E_b/N_0}\right)^2}{2}} dy. \qquad (11)$$

where: $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-\lambda^2/2} d\lambda$.

B. ICTH

For the rate $1/3$ distance optimized code, the first three terms of the expansion are

$$\left.\frac{\partial T_{I3}(X, Y)}{\partial X}\right|_{X=1} = 3Y^8 + 15Y^{10} + 58Y^{12} + \ldots \qquad (12)$$

resulting in

$$[d_{free}, d_a, d_b] = [8, 10, 12]$$

$$[\beta_{dfree}, \beta_a, \beta_b] = [3, 15, 58].$$

The probability of bit error is upper-bounded by,

$$P_b \leq 3P_2(8) + 15P_2(10) + 58P_2(12) + 33P_2(14) + \ldots \qquad (13)$$

In both cases, a lower BER bound may be obtained by considering the first term (free distance) in the expansions.

V. Comparison with Simulations

Fig. 3(a) shows the BER bound considering the first three terms in (10), and the Monte Carlo simulation results for the rate $1/3$ PCTH scheme. Note that the bounds and simulation results converge at high signal-to-noise ratio.

Fig. 3(b) shows the BER bound considering the first three terms in (13), and the Monte Carlo simulation results for the rate $1/3$ ICTH scheme. Note that ICTH exhibits a gain of about 1.5 dB over PCTH @ BER=10^{-3}. This is a direct consequence of the fact that, unlike PCTH, ICTH is based upon convolutional codes with optimized distance properties.

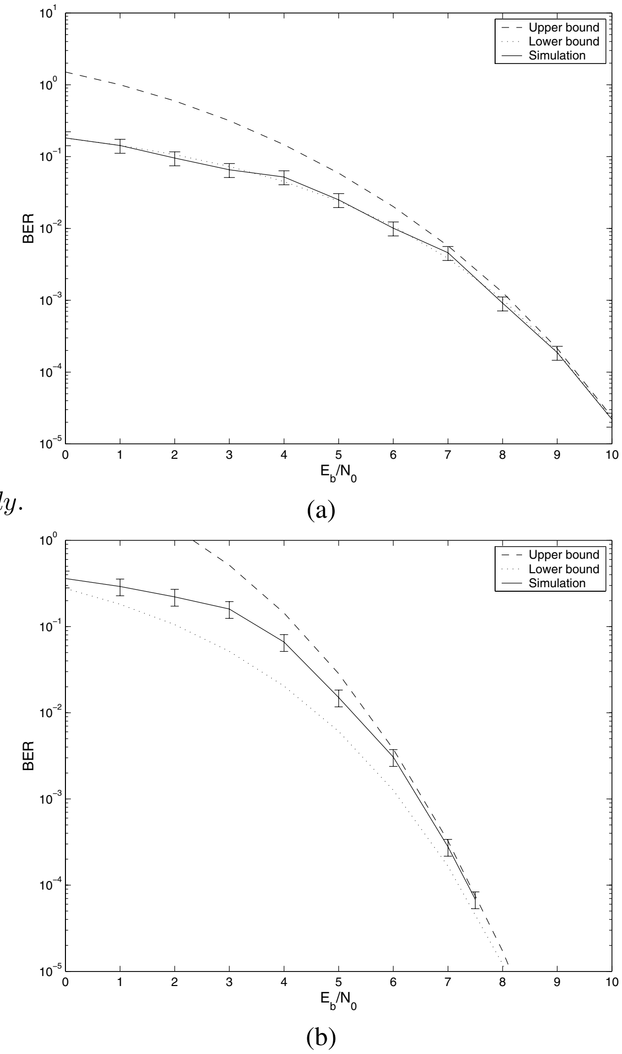

Fig. 3. Simulated vs. analytical BER performance of the a 3-bit scheme using hard Viterbi decoding with an interleaving depth of $L = 18$, in the presence of AWGN: (a) PCTH, and (b) ICTH. Note the ~ 1.5 dB improvement exhibited by ICTH over PCTH @ BER=10^{-3}.

VI. Statistics of the Transmitted Signal and Power Spectral Density

This section deals with the statistics of the transmitted signal, in terms of slot usage distribution, and the corresponding spectral properties. The histogram in Fig. 4(a) shows the relative frequency of occupation of the time slots in the 256-PPM case (n=8) for ICTH, without interleaving. The corresponding PSD (power spectral density) is shown in Fig. 4(b). On the other hand, Fig. 5 shows similar plots for ICTH with $n = 8$ with interleaving.

From Fig. 4, the effect of the structure imposed by the code is visible in the slot usage distribution and the corresponding PSD.The interleaver has a smoothing effect on the spectrum (see Fig. 5(b)) and also the slot usage distribution tends to become uniform, as it can be evinced from Fig. 5(a).

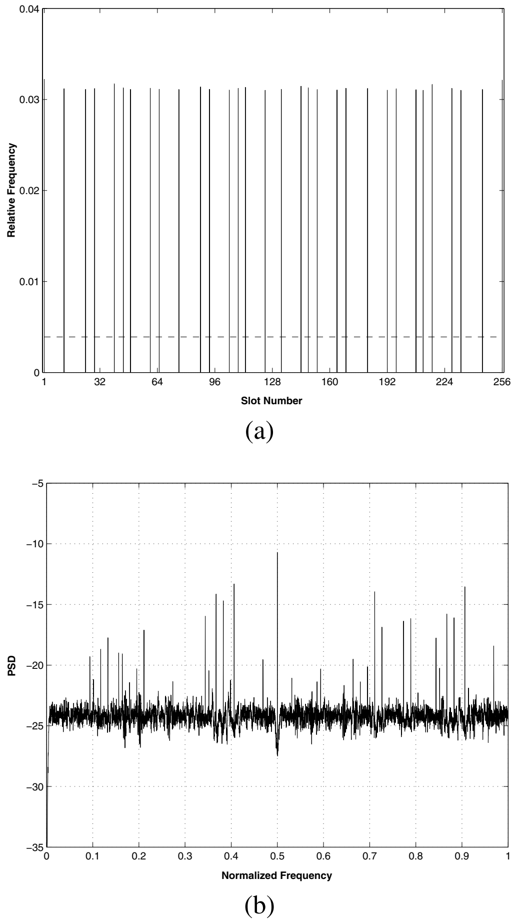

Fig. 4. (a) Histogram showing the slot usage distribution, and (b) PSD of the transmitted signal for ICTH, without interleaving.

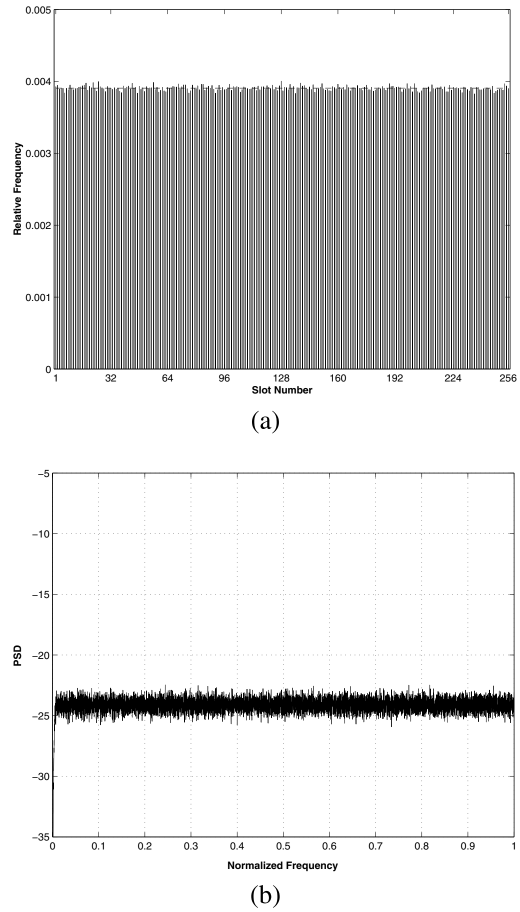

Fig. 5. (a) Histogram showing the slot usage distribution, and (b) PSD of the transmitted signal for ICTH, with an interleaving depth $L = 16$ symbols.

VII. Conclusions

Interleaved convolutional time hopping (ICTH) is a novel coding/modulation scheme for UWB impulse radio, derived from pseudo-chaotic time hopping (PCTH). ICTH relies upon a low-rate convolutional code, with optimized distance properties, and uses BICM. In this paper we developed an expression for the error bound for PCTH and ICTH based on the polynomials describing the codes. ICTH exhibits a significant improvement of the BER performance over PCTH. Moreover, the bit interleaver introduced because of BICM has a desirable smoothing effect on the spectrum.

Acknowledgments

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